The learner as designer

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Processes and effects of an experimental programme

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The learner as designer:
Processes and effects of an experimental programme in modelling in primary mathematics education

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geboren te Utrecht

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CONTENTS

1. General Introduction

This thesis is about modelling in primary mathematics. Modelling refers to the choice or design of models, for example a drawing, a table, a diagram or a formula to represent a problem situation. Modelling can be used as an important step in a problem-solving process, both in its procedural and its heuristic functions. It may be conceived of as a form of strategic learning. The aim of this research project was to obtain greater insight into forms of strategic learning[1]. Our point of departure was a central question in the literature on modelling and strategic learning: is it better to provide pupils with models and strategies, or should pupils learn to generate these models or strategies themselves? (Rosenshine, Meister & Chapman, 1996).

The notion of strategic learning is understood from two perspectives. First, from a longer research tradition in the Netherlands originating in the work of Van Parreren on action psychology (1993), which strongly draws on the works of Vygotsky (Davydov, 1988). Strategic learning is seen as a core element of learning activities that promote human development, and is assumed to contribute significantly to the transfer and meaningfulness of learning outcomes. Strategic learning refers to an activity in which the learner is not just an executor of externally provided actions, but has a say in the planning, execution, regulation and evaluation of his actions. It implies that pupils adopt or adapt a suitable procedure or method on the basis of their analysis of a problem situation. In this context one could think of different approaches, techniques, algorithms or heuristics to solve a problem. As part of such procedures, the heuristic representation of the problem situation by the choice or design of models is considered to be an important element. According to Van Parreren, strategic learning examples are "learning how one solves word problems, writes an essay, understands a difficult text or throws a ball as far as possible" (van Oers, 1996; van Parreren, 1974; van Parreren, 1993, pp. 58, 59, 64 & 71). Characteristically, in these examples there is no unique and fixed method to be acquired in order to reach the goal. The performance always requires exploration to some extent, as well as deliberation on the steps to be taken.

Secondly, Freudenthals' theory of 'mathematics as a human activity' also provided an important perspective for the present study. From this 'realistic mathematics education' (RME) perspective, modelling is seen as a specific strategy within the overarching general strategy of mathematising (Freudenthal, 1991; Gravemeijer & Terwel, 2000; Keijzer & Terwel, 2001).

In Freudenthals' view mathematics should be taught as mathematising. Mathematising literally means 'making more mathematical'. For Freudenthal, mathematics was first and foremost an activity, a human activity. In traditional mathematics education the result of the mathematical activities of others was taken as a starting point for instruction. Freudenthal (1973) characterised this view as an anti-didactic inversion. Things are topsy-turvy if one starts by teaching the result of an activity rather than by teaching the activity itself. [Mathematics as a human activity] is "…an activity of solving problems, of

[1] This thesis is the result of a research project that was conducted within the context of the research programme 'Strategic Learning in the Curriculum', initiated by the Department of Education and Curriculum at the Faculty of Psychology and Education of the Vrije Universiteit Amsterdam.

looking for problems, but it is also an activity of organising a subject matter". (Freudenthal, 1971, pp. 413-414)

Freudenthal uses the word mathematising in a broad sense: it is a form of organising that also incorporates mathematical matter. Instead of transmitting a ready-made mathematical system, modelling might emerge in the process of organising the subject matter (Freudenthal, 1971, 1973, 1991; Gravemeijer & Terwel, 2000).

Teaching children is a highly complex phenomenon . Children should of course learn all kinds of facts, but even more important is that they should learn how to analyse and solve new and unfamiliar problems. The question of how the teaching of more complex problems and strategies should be dealt with in education has lead to several answers. Strategic learning is one of the answers, and in this thesis strategic learning is operationalised as 'modelling the problem situation'.

With regard to the education of complex aims we can generally distinguish two general approaches, a transmission approach and a developmental approach. We will discuss these approaches here, and focus on one strategy to cope with complex problems in particular: a modelling strategy as an integral part of a problem solving cycle. Furthermore, we will show how different strategies fit into these two general approaches.

(1) A transmission approach: in this approach the behaviour to be acquired ("expert behaviour") is taught to the pupils in a direct way, often in a stepwise manner, in which every step instructs the pupils in the acquisition of an element of the final behaviour, or in the acquisition of a simplified version of the final objective. The expert behaviour is typically seen as the final stage in the learning process. With regard to the development of modelling in pupils, this approach in practice often comes down to providing the pupils with the targeted models and explaining to them how they should be applied. After this explanatory (expository teaching) phase with carefully tailored instructions, the pupils start applying their knowledge to a number of especially selected situations ("tasks"), in the mean time practising this knowledge and the associated skills. The pupils (as novices) play a receptive role here; the teacher operates generally on the basis of the well-known Initiation-Response-Evaluation (IRE) format. When the task is too complicated to be taught in one step, the task is often broken down into different component tasks that are then to be integrated into the curriculum in order to achieve the final (expert) goal.

(2) A developmental approach: one of the assumptions behind this approach is that pupils should appropriate desirable activities (e.g. modelling) by getting involved with experts in the target activity. By participating in this activity with the help of experts, pupils perform a precursor of the desired behaviour, which is already part of their acting repertoire. According to this approach, the expert is not just the final stage of the learning process, but acts as a better-informed assistant in the pupils' learning process and helps the pupils to accomplish the desired activity in a form that reflects an acceptable version of the expert's activity. For example, the expert takes over the aspects of the activity that the pupils cannot carry out for themselves. In most cases this means that the teacher (the expert) asks critical questions about the created model, reflects on its consequences, aligns and confronts it with other models, etc. (in the same way as would be done in the community of experts working on the construction of a shared

model). The learning process here is not the adoption of an external model or a model construction procedure, but *the transformation* of the pupils' own models and ways of model construction in a developmental process. The pupils take on an active designing and constructing role, and are involved in discourses with one another and with the teacher. The teacher is not the provider of models, nor the controller of the learning process, but a critical participant in the learning activity. Learning here is essentially a process of co-construction of models and of modelling strategies. The theoretical perspectives, in this case Van Parreren's work on action psychology and Freudenthal's work on teaching methods are, broadly speaking, examples of a developmental approach.

Research Question

Through the use of contexts (situations and activities from the daily-world of pupils related to open, complex problems), educators try to make education more attractive and meaningful to pupils. The use of contexts may also promote the application of acquired knowledge in new and unfamiliar situations. However, research has shown that this is not always the case as pupils sometimes do not have access to the skills required to solve complex tasks in contexts. There has been a growing interest in strategies for the solving of less-structured tasks, with an eye to transfer (Rosenshine, Meister, & Chapman, 1996). Until now, little research has been done into strategies for solving open and complex problems in contexts (Hattie, Biggs, & Purdie, 1996; Hoek, Van den Eeden, & Terwel, 1999; Rosenshine et al., 1996). How can pupils obtain support from being familiar with these strategies? Here, Rosenshine et al. raise an important question: is it better to provide pupils with models and strategies, or should pupils learn to generate these models or strategies for themselves? In fact, the two approaches form a contrary pair here: providing versus generating (note the similarities with the two educational approaches we discussed earlier, namely, the transmission approach and the developmental approach).

However, Rosenshine et al.'s study fails to mention an approach that is even more interesting in our view. We decided to leave the contrary pair 'providing versus generating' aside, and instead to focus on a new contrary pair: 'providing versus designing in co-construction'. The main research question posed in this study is the following:

> What are the effects of an experimental learning-in-context programme – in which learners are seen as designers – on the learning processes and learning outcomes of pupils in primary mathematics education, compared to the learning outcomes of pupils in a control group in which ready-made models are provided by the teacher?

Hypothesis

In this experimental programme, defined as 'designing in co-construction', teacher and pupils together design and elaborate models and strategies in the process of solving complex tasks. The following hypothesis was formulated: the designing-programme is especially promising for the promotion of transfer. Pupils who learn how to model and represent complex problems, may arrive at better results when confronted with new and unfamiliar tasks than pupils who have been provided with models.

Learning to design will lead to a deeper insight into principles or conceptual structures behind tasks and that will presumably facilitate learning and especially transfer, as was measured by the posttest and the transfer test.

With respect to the learning outcomes from both programmes we expect that differences will be found in the transfer of the learning outcomes to new, complicated situations. Due to the active involvement of the pupils and the fostering of insight in a constructive approach to problems via model construction, we expect that pupils from the designing condition will be better equipped to solve the complex problems on the transfer test.

Research Design

The conceptual model guiding the study is mapped out in Figure 1. We can describe this model as follows. Pre-knowledge in mathematics is transformed by learning processes into learning outcomes. It was hypothesised that the experimental programme accelerates this process as compared to the control programme.

In order to examine the role of strategic learning in mathematics, we conducted a series of studies in one of the upper primary grades (grade 5, ages 10 - 11) at several schools. First, we deliberately used a case study design for the first study. Secondly, on the basis of the knowledge gained from the case study a larger experimental study was conducted. This larger study was a quasi-experimental pretest-posttest design, in which 10 classes, 238 pupils and 10 teachers participated. Using the data from the larger experimental study a series of specific studies was carried out.

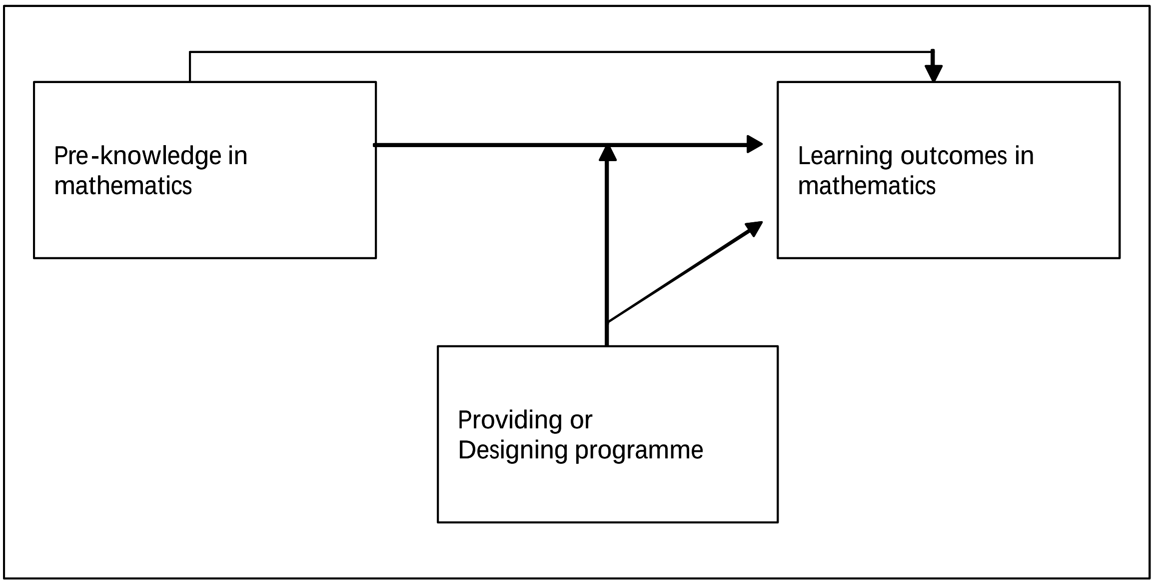

Figure 1: Conceptual model guiding the study

An intervention was developed by constructing two prototype programmes in which two conditions were operationalised: a providing and a designing condition. Both programmes have the same mathematical content. The difference lies in the fact that in the designing condition explicit attention is given to the designing process, which means representation, modelling, elaborating, and recontextualising. The tasks and assignments to be done in the lessons were open, complex problems. These were chosen from current maths textbooks used in the Netherlands (Pluspunt, Wereld in Getallen Nieuw) and in the United States (Mathematics in Context). Also, exercises were used which had been developed for MILE (Multimedia Interactive Learning Environment), a project of the Freudenthal Institute for Teacher Training. All the exercises used were adapted to fit the purposes of the intervention intended. In addition, a teachers' manual was developed containing the main ideas and principles of the two programmes[2].

The intervention consisted of a one-hour lesson every day for (nearly) three weeks. It consisted of 13 lessons: one lesson in which the children learned about strategy use, models and their functions, and 12 lessons on percentages and graphs. Four lessons dealt with percentages, another four lessons dealt with graphs. In the rest of the lessons, percentages and graphs were combined.

In the designing process, which is a process of developing models in co-construction, both the teacher and peers play an important role. The joint activity functions as a configuration that gives the participants several opportunities for help and support. In this way, the joint activity turns into distributed cognition: for each participant in the activity resources are available, offered by other participants (including the teacher), as well as instruments. Consequently, each participant is more capable than he or she would be as an individual.

<u>Research Situation</u>

The pupils in this study were 10 – 11 year old children in the fifth grade of primary education. They were situated in ten classes. These classes were matched, according to geographical location, mathematics method used and school population. Five of the classes were assigned to the control condition, the other five classes participated in the experimental condition. All pupils were introduced to a series of lessons in which they learned about percentages and graphs. Both conditions followed the same structure. Informal knowledge and experiences of pupils were taken as a starting point to explore the subjects of percentages and graphs. Familiar contexts like the cinema, sports tournaments and shops were used to allow pupils to make connections with their already existing knowledge. From the beginning, connections were made with other learning strands like fractions.

In the providing condition pupils learned to work with percentages in ready made models like pie-charts and bar models. In the designing condition these models were not provided, and models had to be designed by the pupils from the contexts given.

Although the pupils had some earlier experiences with reading and interpreting graphs in grade 3 and 4, and although conditions for percentages in terms of fractions were already met in grade 4 and the start of grade 5, the subject of percentages itself was completely new at the time the intervention started. No formal instruction on the subject of percentages was given until the intervention started, although of course some pupils already had a notion of what percentages meant. In most mathematics methods

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[2] More detailed descriptions about the teacher manuals, programmes and tests can be obtained from the author on request.

formal instruction on this subject is scheduled to start in autumn. In the one case formal instruction on percentages was scheduled earlier in the learning trajectory, the teacher was asked to skip the subject and to wait for the intervention to start. This way the starting points of all pupils were kept as equal as possible. Models were not totally new either, the pupils did work with some models in advance, for example with pie-charts and bar models for the subject fractions.

To determine the pre-knowledge in mathematics of the pupils before entering the experimental curriculum, a standardised National mathematics test was administered before the start of the intervention. Afterwards a curriculum-specific posttest and a special designed transfer test were administered. Both tests contained an equal amount of percentages tasks and graphs tasks. In a couple of other tasks, percentages and graphs were combined. The posttest and transfer test were administered shortly after one another.

The curriculum-specific posttest consisted of tasks highly comparable to the tasks to be done in the lessons, and they were of the same difficulty level. These tasks asked for strategies as identifying percentages in a given context, shading an amount of percentages in a model, calculations with percentages, reading graphs, interpreting graphs, and drawing information in graphs. All knowledge tested in this posttest was explicitly taught during the intervention lessons in both conditions.

The transfer test, however, consisted of tasks that were not so closely related to the tasks practiced in the lessons. The tasks were of a higher difficulty level and asked for insight, concepts and strategies that went beyond the content of the lessons. For example, pupils were confronted with the concept of permillages, or with three-dimensional percentages problems. In order to solve these tasks correctly pupils needed to reformulate their ideas and concepts. Some of the assignments in the transfer test were accompanied by a request to show the model that was used to solve the task.

The posttest consisted of 22 items, partly open and partly closed, with an alpha of .83. Pupils could earn points for every correct answer. The open questions, and some of the other tasks required an estimation or an explanation. In that case pupils could score a certain amount of the maximum score if they made the estimation between certain limits, or if they mentioned the most important elements in their answers.

The transfer test consisted of 17 partly open and partly closed tasks. Regarding the open tasks, pupils could earn scores depending on their answers, as was also the case in the posttest. If they mentioned the most important elements they were given the maximum score. The alpha of the transfer test was .76. The scoring of the transfer test was carefully controlled for bias by the researcher through the employment of a second judge and by using a blind judgment procedure (see for details chapter 4).

Outline of the Thesis

From several experimental studies it appears that providing pupils with ready-made models is more effective than allowing them to generate models themselves, although Rosenshine states that further research into this topic is needed: studies that compare the effect of providing strategies to pupils with asking pupils to generate their own strategies would appear to be worthwhile (Rosenshine et al., 1996).

There is still discussion about the relative value of either of these approaches for educational practice. Several researchers advocate the providing approach, for example Hattie et al. (1996), Mayer (1989), and Perkins and Unger (1999). Others advocate the designing approach, for example Davydov, (1988), diSessa, (1991), Meira, (in press), and van Oers, (in press). It is hoped that, as a result of this thesis, new light may be shed on this ongoing discussion and that new input may be brought to it.

In this thesis, the results of the studies referred to above will be mapped out in several chapters. Because each chapter is a complete article written for different international journals, some repetition among the chapters is inevitable[3].

The present work started with a case study, allowing an exploration of both conditions. Following this introduction, the small-scale study is described in chapter 2. Two pupils are followed in depth, each pupil coming from a different classroom, and examples are shown of the tasks and models the pupils worked with. The preliminary results of this in-depth analysis are subsequently used for the following chapters. We expected that pupils in both conditions would score equally well on the posttest, but would differ significantly on the transfer test. It was therefore decided to work out these hypotheses in more than one chapter. In Chapters 3 and 4 the effects on the learning outcomes of pupils in the providing and designing condition, concerning respectively the posttest and the transfer test will be examined and discussed. The question remained of what processes occurred in the designing condition, and how that might affect the development of models. Chapter 5 analyses in greater detail the processes that occur during the intervention in the designing condition. On the basis of a theory of levels in the learning process, special attention is devoted to the transition from one level to a higher level. In the last chapter the preceding chapters will be connected, followed by a general discussion of the research project. A summary in Dutch completes the thesis.

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2. Providing or Designing? Constructing Models in Primary Maths Education[4]

[4] Van Dijk, I. M. A. W., van Oers, B., & Terwel, J. (in press). Providing or Designing? Constructing Models in Primary Maths Education. Learning and Instruction.

Abstract

The goal of this exploratory study was to uncover the construction processes which occur when pupils are taught to work with models in primary maths education. Two approaches were studied: 'providing models' versus 'designing models in co-construction'. A qualitative observational study involved two groups of pupils of a primary school in the Netherlands. A series of lessons involving problem-oriented tasks was given. For this article's purpose, we studied the learning processes of one pupil per condition in detail. Interpreting the results it is assumed that upper-grade pupils may be able to design models in co-construction as long as they receive sufficient teacher guidance.

Introduction

The issue of model use and construction in primary school pupils has got wide attention over the past twenty years (see for example Gentner & Stevens, 1983; Giordan, 1988; Seeger, 1998; Vosniadou, 1994). Despite the variety of definitions of what a model is, we can conclude from the literature that an element of structured, symbolic representation is common to all definitions of models used as tools for problem solving for the pupils. In the present article we will use the term 'model' in a similar way.

An important question is whether it is better to provide students with ready-made models or whether they should generate such models for themselves. Rosenshine, Meister and Chapman (1996), for instance, pointed to studies suggesting that provided models are more efficient than self-generated models (i.e. models designed by the students themselves).

The question of whether models should be provided to or created by students remains unresolved. In a theoretical analysis of this dilemma, Gravemeijer (1997) points out that introducing models to students actually boils down to the straight transmission of those models and the meanings that adults or experts have already associated with them. On the other hand, creating models implies that students have to invent something new: they have to produce models as emerging from their own activity. In research conducted by Mayer (1989) positive effects were found of providing pupils with teacher generated models. From these research findings, we questioned whether the dilemma 'providing or generating' could in classroom practices be seen as a realistic opposition, as students never create models 'out of the blue'. Even in situations which allow them a large measure of freedom, students always obtain some help and hints from the communicative setting in which they are engaged. Moreover, in the context of our research program we are especially interested in 'guided reinvention' of models in mathematics education (Freudenthal, 1991; Gravemeijer & Terwel, 2000). We therefore reasoned that a more appropriate dilemma to be studied was the opposition between the learning processes based on providing ready-made models versus teaching how to collaboratively design models. The latter is conceived as a teaching / learning activity in which teacher and pupils interactively design representational tools. A balanced combination is made here between the models suggested by the teacher and the pupils' own productions, on the basis of a negotiation of the meanings of those pooled models. We emphasise here that in the present article the notion of a model should be conceived as a representational model which functions in the broader context of a problem solving strategy. It could be a drawing, a diagram, a scheme, an equation, or some other form of symbolic representation. In the present study we focus particularly on a qualitative, exploratory analysis of differences in problem-solving processes for percentage problems, in which the model was provided by the teacher in one group, while in the second group pupils were stimulated to design a model for themselves in co-construction with peers, and under the guidance of the teacher. The former of these conditions represents the traditional standard-type of teaching, the latter is an attempt to create an innovated socioconstructivist way of teaching, along the lines of Realistic Maths Education. The research questions concern how pupils cope with mathematical problems under the two conditions and how they choose, use and construct models. Main questions in this article are:

1. How do the learning processes of pupils proceed when models are provided by the teacher as an aid for the solving of problems in mathematics?
2. How do the learning processes of pupils proceed when pupils collaboratively design models, guided by the teacher as an aid for the solving of problems in mathematics?
3. Are there differences in the way the pupils deal with models in the two conditions: ‘providing’ versus ‘designing’?

The present analysis was a first step in our research project. We expected that this could lead to a deepening of our understanding of the factors facilitating transfer, and as such to a better starting point for the design of our main study into model construction and transfer. In this initial qualitative study we expect to find that upper-grade primary school pupils are capable of designing, applying and improving models in interaction with peers and the teacher when working on percentage tasks.

Theoretical Framework

The theoretical and empirical basis of the present study corresponds to two perspectives: (I) the sociocultural perspective of Vygotsky and its further developments (Rogoff, 1990; Wertsch, 1985), and (II) theory and research into realistic mathematics education inspired by the work of the Dutch mathematician Freudenthal (1991) (see also Cobb, Yackel and & McClain, 2000; Terwel, 1990).

The neo-Vygotskian approach conceives of learning as a relatively permanent change in the actions of an individual. These changes manifest themselves with regard to the structure of the actions involved and with regard to the meaning of those actions (Freudenthal, 1991; van Oers, 1990, 1996; Rogoff, 1990; Wells, 1998; Wertsch, 1985). In cases where learners themselves are reflectively involved in creating the conditions that can produce these changes, we talk of ‘strategic learning’. Here learners act as agents in the learning process and are not simply objects to be modified by instructional procedures. Learners plan, monitor and evaluate the course of their actions and learning process, but also construct their own tools for the regulation of the learning activity. Joint activity, exploratory talk and discourse are seen as important elements in this approach to teaching (Forman et al, 1998; Driver et al, 1998)

On this point we can recognise substantial similarities with the theory of realistic maths education of Freudenthal (1991). For Freudenthal, mathematics was in the first place an *activity*, a human activity in a real life situation. He advocated active learning of mathematics in small, heterogeneous groups under the guidance of the teacher (guided reinvention). Doing mathematics was more important to him than mathematics as a ready-made product. And, according to him, the same should hold for mathematics education. It is the doing of mathematics that leads to the result of mathematics as a product. In traditional mathematics education, though, the result of the mathematical activity of others (!) is taken as a starting point for instruction.

Important and obvious questions at this stage pertain to the aspects of the actions that need to be reflected on, as well as to the role of others (peers, teacher, parents) in the learning process. The cultural-historical model of human activity refers to several elements of paramount importance: the object (what should the actions be directed towards?), the goal (to what end should the object be transformed?), the motive (why should the action be carried out?), and the tool (which instrument should be used to change the object according to the goal?). The tool to be used in the process of acting is seen as a key element in the process of acting.

In the course of our cultural history, tools have undergone a number of special developments. In the intellectual domains, a special category of tools has been developed which embody a great deal of knowledge on how to cope with special problems in the discipline. These tools are called ‘models’. As embodiments of cultural knowledge they provide effective means for the regulating of one’s own actions and the actions of others. It is widely argued nowadays that the development and use of theoretically and scientifically acceptable models is necessary for the development of thinking. The use of abstract models is believed to produce better transfer of previously learned material. At this point, both the cultural-historical (sociocultural) and the ‘realistic mathematics education’ (RME) point of view emphasise the importance of models in the processes of learning and problem solving (Cobb et al, 2000; Davydov, 1988; Freudenthal, 1991; Gravemeijer, 1997; Hedegaard, 1990; Van Oers, 1996; Säljö & Greer, 1997).

In recent elaborations of these approaches, pupils’ reflections and discussions on the shared model-making process are now generally recognised as a major element in the learning process. Participation in the process of model design in collaboration with teacher and peers is considered to be highly relevant to the development of pupils’ learning in a particular area of knowledge. Within this point of view, model design mainly consists of the construction of mental objects that have acquired special instrumental meaning in a community of practice (see Sfard, 1999). The process of meaningful model construction is closely linked with the conception that views model formation as a social activity based on the co-constructive production of tools for thinking.

Although this view of modelling as a co-constructive process is widely accepted by many cultural-historical and socioconstructivist educationalists, it has not been universally held to be self-evident, not to mention that the learning effects are disputed. Rosenshine, Meister and Chapman (1996), for instance, have explicitly addressed the question whether it is better to provide students with models or whether they should generate such models for themselves. These authors pointed to empirical studies that may be interpreted as suggesting that provided models are more efficient than self-generated models (i.e. models designed by the students themselves). However, Mayer also found that high-aptitude students who where provided with models did not perform better than high-aptitude pupils in the control group on problem solving. High aptitude pupils come to the lesson with their own models (or the ability to rapidly design them). For these pupils the provided models are probably too simple and in conflict with their self-designed models.

The present study and its research question should be conceived in the context of this theoretical background. The overarching research question is how pupils cope with mathematical problems under the two conditions (providing versus designing) and more specific how they choose, use and construct representational models as a key element in a problem solving strategy in primary mathematics.

On the basis of findings by Davydov (1988), Landa (1999), Boekaerts and Simons (1993) and others, it may be expected that pupils who learn to design models:

1. will be more active and more goal-oriented in carrying out their tasks;
2. will acquire a more flexible organising tool for learning;
3. will be better able to deal with new and unfamiliar tasks.

Since we have opted for an exploratory study in which the processes taking place are more directly taken into account than the factual learning outcomes, these expectations cannot rigorously be tested in this part of the project. However, in combination with the videotaped lessons, the completed tasks gave an impression of how the pupils dealt with (provided or designed) models. Hence, we expect to find information that helps us evaluate the plausibility of the first two expectations.

Methods

Design

Two grade-5 teachers, both working at the same primary school, implemented an intervention programme, which consisted of a series of lessons on percentages, developed according to the two different approaches mentioned earlier. One teacher and his group of pupils followed the ‘providing’ version, in which the teacher provided models and offered whole class instruction and guidance in the use of these models. The other teacher and his group followed a similar programme in which emphasis was placed on the guided designing of models in co-construction by the pupils and teacher. The school was selected on the basis of previous research contacts and their willingness to participate in the experiment. It is a public school which is situated in a village near one of the larger cities. The student body comes from a predominantly white middle class community. Thus, the school is not an ‘average’ school and the descriptions cannot be generalised over all kinds of schools. However, we considered the comparison between the two classes (conditions) as possible because of the similarity of the two conditions on several points (school population, mean ability score in mathematics, preparation of teachers for this research, and previous mathematical content).

On the basis of a case-study design (Yin, 1989) we studied differences in processes throughout the interaction in the two conditions by means of direct observation and descriptive analysis of videotaped lessons. The differences in solution methods were studied by analysing the teaching materials completed by the pupils. We checked whether the pupils used models (either provided or designed) and, if this was the case, what kind of models they chose to work with and how they were used in the problem-solving process. The development of the pupils in each condition was followed throughout the five lessons. The aim of the study was merely to gain an understanding of the processes that occurred in the two conditions and to compare them.

Curriculum Materials

The curriculum materials (programme) developed for this purpose consisted of five lessons. The first was an introductory lesson to discover what preconceptions the children had and to explain the use of models in mathematics. This was followed by three lessons on the subject of percentages in which models were either provided by the teacher or designed in co-construction, depending on the approach in question. In the closing lesson the pupils conducted their own small research project and presented the results using percentages and models. In these lessons, models were regarded as an organising tool for learning. Both versions consisted of the same (paper and pencil) tasks: a combination of less-structured percentage tasks in real life contexts, selected from Dutch maths courses. The assignments in the two versions of the student materials differed in terms of the written instructions given. The group of pupils following the 'providing' lessons were provided with a limited set ready-made models to choose from, while the 'designing' group of pupils were stimulated to collaboratively design their own models.

Percentage problems were chosen as main topic in our research because this was a new area for the students which enabled the researchers to analyse modelling processes in a new domain of mathematics. Pupils did not have any formal instruction in percentages, which ruled out possible interference from previous teaching on this topic. However, pupils had some informal knowledge and experiences regarding percentages.

Instructional Support

The core of the difference in the instructional support concerns the fundamental distinction 'providing versus designing' while holding other factors constant as much as possible. Both classes received whole class instruction and guidance by the teacher how to use a problem solving strategy (a heuristic approach) in similar contexts. As a part of this strategy the choice and use of a model from a limited set of teacher provided models (pie, bar etc.) was the core of the support in the providing condition. By contrast, the teacher support in the designing group was primarily directed to demonstrate and support the design process: how to make your own representations of the problem situation by using a variety of own ideas and productions (representations) ranging from a drawing, a story, a scheme, table, concept map etc. As the experiment took place in real classroom settings, it has to be mentioned that there were some intrinsically related differences in the instructional support which belong to the specific condition whether providing or designing of models is concerned. While the teacher in the experimental group was focused on the design process, the control teacher was especially directed to providing and use of the ready-made models. It doesn't mean that children in the providing condition received less help and teacher support than their counterparts in the designing condition. In class-wide discussions, instruction and personal hints these children were also encouraged to improve their use of (provided) models.

In the designing condition, a diversity of models was expected to emerge. These models were not yet expected to be completely mathematically correct, but to correspond to the thinking levels of the pupils who made them and therefore to form an appropriate starting model ('model-of', Gravemeijer, 1997). It is important to note that hints from the teacher and discussion with other pupils are essential for improving the self-designed models. In this regard we can speak of increasing formalisation: from drawings, to diagrams or schemes and on to more generally applicable models ('model-for', Gravemeijer, 1997).

Sampling, Data Collection and Analysis

For the purpose of this article we selected two children: Sharon (providing condition) and Donja (designing condition). The two pupils were comparable in respect to age, background and achievement in mathematics. Both pupils are 10 years of age, having a Dutch white middle class background. According to the teachers the girls are also comparable in other relevant dimensions like motivation and general aptitude. The teachers judged Sharon and Donja to be average pupils in mathematics and this was confirmed by their scores on a national standardised maths test taken a month before the start of the study. In this test, the scores are converted into a national standard level, and divided in five levels: A (very high) to E (very low). Sharon and Donja scored on the first part of this test respectively 66 and 64 points. On the second part they scored respectively 64 and 65 points. On the scale from A to E, they both score a C.

After each lesson the completed assignment books were collected. We followed the children from each condition in their development, by analysing the videotaped lessons and the completed tasks. During each assignment, the actions undertaken by the pupils and their choices in the use of models were observed in detail. After each lesson, the students were interviewed to find out what they had been thinking while completing the tasks. These sessions were assimilated in the interpretation of the videotaped lessons. The collected materials consisted of two types of data: videotaped lessons and the tasks completed by the pupils. The tapes were transcribed in protocols, which formed the basis of an analysis for the purpose of this article.

Results

As was formulated in the very first paragraph of this article, we expected children in the upper grades of primary school to be capable of designing, applying and improving models when working on percentage tasks, in interaction with peers and their teacher. This expectation did not appear to be borne out when we studied the first lesson on the subject of percentages in the design condition. The children did not know what to do with the freedom given to them, even though this lesson was preceded by an introductory lesson about models. They seemed to be afraid to commit their ideas to paper and if they did, they erased it or dismissed it very quickly afterwards. However, during and after the first lesson some primitive models did emerge which were improved in interaction with the group and the teacher.

To give an impression of the development of pupils during the lessons and to answer the question on the course of learning processes stated at the start of this article, we would like to describe and analyse the work of Sharon (from the providing condition) and Donja (from the designing condition).

In this article we will focus on some of the tasks both Sharon and Donja completed during the three lessons on percentages. It supports our assumption that the two conditions do indeed differ with regard to the ways in which pupils deal with models.

<u>The First Percentage Lesson</u>

Providing Condition

In the first percentage lesson the concept ‘something from hundred’ was introduced. Assignment 2 of this lesson is an exercise about two kids playing darts. The text was introduced as follows:

> Evelyn and Peter brought a dartboard from home. They can both play quite well. They were showing off about how good they are. Evelyn says: ”When I throw a series of 20, I usually score 4 bull’s-eyes”. Peter thinks he’s a better player. He says: ”When I throw a series of 25, I usually score six bull’s-eyes”. The children standing around them butt in on the discussion. One girl thinks Evelyn is a better player, while another girl thinks that Peter is better. What do you think?

In the providing condition, this problem was first dealt with at class level. The models provided (circle diagrams) were explained by the teacher. The discussion showed the pupils how to use the models and carry out the task. All the children had to do was complete the assignment in the way the teacher showed them. Sharon had no difficulty filling in the circle diagram afterwards. She forgot to fill in some of the proportion tables in her assignment book because the teacher had already done them on the blackboard.

Designing Condition

The task was more difficult for Donja, who took part in the designing condition. The group she worked with had to discuss the assignment without a thorough previous instruction, as was the case in the providing group. Donja felt ill at ease. The pupils were used to being instructed before starting a new task. The interaction between the participants in the group (made up of three girls) is therefore unstructured.

Richelle:	Well, Mirel, what will the answer be? We are allowed to do it together (Talks to Donja, who immediately starts to write).
Donja:	I just do minus (Writes down: “20-4=16, 25-6=19”).
Richelle:	4 out of 20, that needs 5 throws, here 4, well, that’s the …4-times table, yes.
Teacher:	You ought to make some notes.
Richelle:	Yes, yes. Quiet. Here, 5, 5 throws needed. (M and R write down: ‘5 throws needed’)

Mirella: Isn't it just 4/20 and 6/25?

Richelle: Yes, five by four, something from 20, is 5 each time. That is OK as well.

Mirella: I think Peter is a better player, isn't he?

Donja: Yes, I think so too! (writes down: 'Peter is better').

Mirella: I am going to cross this out (deletes '5 throws needed'. Writes below it: 4/20 and 6/25).

Richelle (looking at Donja's paper): Peter is better.

Donja: I think so too. But I did it another way.

(Richelle writes down: 'Peter is better').

Donja: I just did 20 minus 4 and 25 minus 6.

Mirella: I will do it this way. Look here, (???) 6/25 (deletes 4/20 en 6/25 immediately afterwards. She looks at Donja's work. Then she writes down: '20-4=16 and 25-6=19. I think Peter is better'.)

Donja: Really, Peter isn't.. eh, he is better. They aren't equally good.

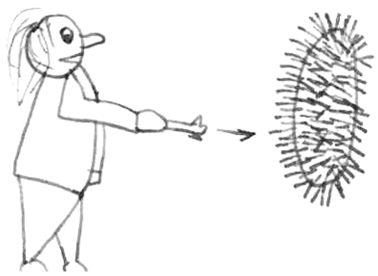

Figure 1: Donja, designing condition

The hint about drawing the situation did not help Donja at first. In an assignment in which she had to represent the score of a person who throws a series of 100 and scores 100 bull's-eyes, she eventually succeeds in designing a primitive model (see Figure 1).

<u>The Second Percentage Lesson</u>

First Assignment, Providing Condition

In the second percentage lesson it appeared that Sharon could not yet properly employ the bar model. In the first task she had to choose one or two pictures out of three and to represent them in a model. Sharon started with an task, which says: 'this strawberry consists of 50 per cent water'. This task turned out to be the only one she managed to complete in the given time (Figure 2).

Figure 2: Sharon, providing condition ('strawberry', 'half', 'fruit-water')

> Sharon draws a bar, divides it into two rows and three columns, like a proportion table, and writes 'strawberry' above the bar. She adds some numbers: in the first row she writes down '50 % and 100' and in the second row she writes '100 % and 100'. A moment later, she crosses out the '100 %' and makes '5 %' of it. Because this model apparently gets her no further, she decides to draw a circle. She starts dividing it into parts, but does not finish the job. Again, she crosses out the circle and thinks for a while about another way to represent the strawberry. Finally she draws a second circle, divides it into two sections and writes 'fruit' in one section and 'water' in the other.

Although Sharon shows a slight improvement in the use of the circle model, it can still be characterised as a way of 'trial and error'.

First Assignment, Designing Condition

Donja, who took part in the designing condition, first said that she did not understand the task. The giraffe assignment said: 'I had a flu, but I'm not for 100 % recovered yet'. Richelle explained to Donja that she just had to make a drawing and showed her drawing to her: a giraffe head with his tongue hanging out of his mouth and sweat-drops on his face. After some thought Donja managed to draw an acceptable model from which to start. She represented this giraffe assignment in an upright bar, like a thermometer.

This designed model was also used to represent the two other assignments. Even though the model she designed was not entirely complete or entirely mathematically correct, it is indeed a good starting point for class discussion (see Figure 3). The teacher noticed her efforts and asked her to explain her model to the class on the blackboard. While she explained her ideas to the class, the teacher emphasised parts of her model.

Teacher: Donja, would you place your designed model next to Maya's? Thanks, Maya, for your effort, it's alright. It's a matter of trial and error and essentially everything you think of is fine. If you are able to explain your thoughts, then eventually it will come by itself.

(Donja draws a big upright bar on the blackboard. Underneath it she writes 'ill' and at the top she writes 'well'. Slightly above the middle of this bar she places a horizontal wavy line, which divides the bar into two uneven parts. Lastly she calibrates the right hand side of the whole bar.)

Teacher: Thank you, Donja. Who else came up with something like this?

Child: I made exactly the same!

Teacher: See, you have placed the giraffe in a bar. This bar represents the giraffe, (points to the bar) and this bar represents how well the giraffe feels. It doesn't matter if you put the line here or there (points at two of the dashes). In what case would this line not be placed correctly?

(…)

Teacher: Is this OK? If I put the wavy line here? (he places a line close to the bottom of the bar, near to the word 'ill')

Child: No!

Teacher: Correct, because that would mean that he's almost recovered. (rubs out the line)

Richelle: No!! But that line there would mean he's dangerously ill!

(The teacher has apparently interpreted Donja's bar upside down. Donja walks to the blackboard to explain the misunderstanding).

Donja: If you put the line here (points to the bottom of the bar) it means that the giraffe is ill, and if the line is placed here (points to the top of the bar) the giraffe is well.

Richelle: Yes, sir, we understand it.

(…)

Teacher: He doesn't feel 100 % better, but… Would that be more or less than 50 %?

Child: More.

Donja: It should be here (points somewhere below her wavy line, but still in the upper half of the bar).

Teacher: Yes, it should be placed in the upper part of the bar.

Figure 3: Donja, designing condition ('The giraffe is a little bit better but not totally recovered yet')

Second Assignment, Providing Condition

At a later stage in the same lesson, Sharon chose an advertisement to represent: a tube of toothpaste with '50 % extra free'. The words Sharon wrote down in her book indicated that she understood the underlying idea: '50 per cent extra' means that, of the amount a tube would normally have contained, an extra half has now been added for free. She understood the message of the advertisement, but unfortunately she did not correctly represent it in her model. She made the mistake of not starting from 100 % and adding 50 %, making 150 % altogether. Instead she started with 100 % and divided it into two parts of 50 % each: so it stayed 100 % altogether. She thought of it as a 'part - whole' problem while in fact it was an ‘increase - decrease’ problem. She did not use the provided bar model in the way for which it had been intended and provided, although the teacher showed the use of it on forehand (see Figure 4).

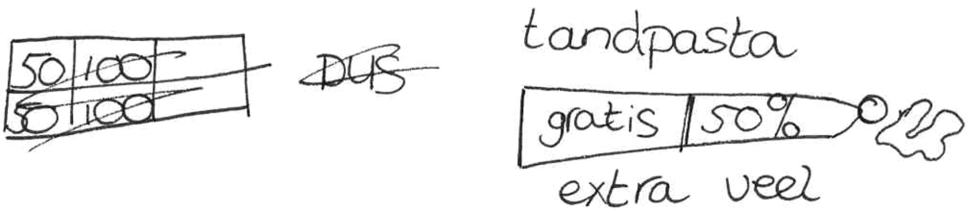

Figure 4: Sharon, providing condition (‘toothpaste, free 50%, extra amount’)

At first Sharon draws a bar, with which she makes a proportion table. Pretty soon she abandons this model and crosses it out. Again she draws a bar, that ends in something like a point. She divides this bar into two equal parts. In the left part she writes ‘for free’ and in the right part she writes ‘50 %’. Above this bar she writes ‘toothpaste’ and below the drawing ‘extra amount’. Her model remains directed towards the material context of the toothpaste, perceptible in the drawing of the cover and some toothpaste lying outside the tube.

Second Assignment, Designing Condition

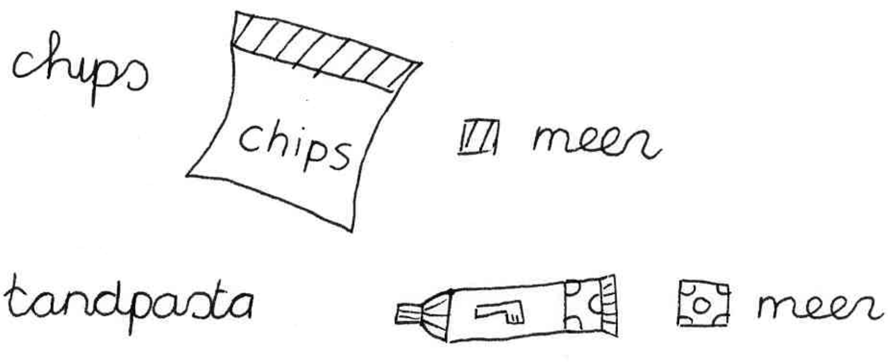

Figure 5: Donja, designing condition (‘chips, more’; ‘toothpaste, more’)

Donja also drew the toothpaste advertisement. While she drew (see Figure 5) it was not possible to see whether she was drawing 50 % of the amount above the ordinary amount of 100 %, or whether she first drew the whole bar and then selected a part of that 100 %, as Sharon had.

> Donja writes down 'toothpaste'. With attention to detail, she draws a special type of bar: a tube of toothpaste. She shades part of it, approximately one-third of the bar, with dots. Next to this tube she draws a square having about the same size as the shaded part of the bar. She also shades this square with dots, and writes beside it, like a key, the word 'extra'.

The model she drew in her assignment book remained strongly related to reality, as was the case with Sharon; both girls drew covers for their tubes. But at a later stage in the lesson during a class discussion, Donja claimed to understand the difference between '50 % extra' (increase - decrease problem) and '...50 % of...' (part - whole problem). Donja showed the class on the blackboard how one should draw a 50 % increase in an amount. The following part of the lesson gave us reason to believe that she knew this difference while drawing her model of the toothpaste.

(Donja places her finger exactly halfway along the bar and judges how long this half-tube (50 %) is. She then extends the bar by this estimated length).

Teacher: Very good.

(Donja shades the part of the bar she just extended. Then she makes a key and writes next to it 'extra'.)

Teacher: OK, thank you. Look at this one. The previous ones are not important right now. Focus on this one (points at Mirella's model) and this one (points at Donja's model). Can you tell me why this one is correct (points at Donja's model), and this one isn't (points at Mirella's model)? Anyone? Peter?

Peter: Well, that one (referring to Mirella's model) is an ordinary tube of toothpaste, and it just said something like… Look, that's not 50 % extra, so you have to add 50 % extra to it, attached to the tube.

Teacher: So what needs to be larger?

Peter: The tube.

Teacher: Yes. Look at this, Mirella. When you walk into the shop, and you see this tube of toothpaste (draws a bar) and next to it a tube with '50 % extra toothpaste' (draws a second bar of the same size), could the 50 % extra toothpaste have fitted into the ordinary tube?

Richelle: I would take that one.

Children: No, it couldn't.

Teacher: Right, if you get half of the usual amount extra, it won't fit into the original size tube. So you have to make the tube larger. What Donja did was right, attaching a piece of tube to the original tube, which she estimated very well.

The Third Percentage Lesson

Providing Condition

In the third percentage lesson, Assignment 1 is about ‘how many per cent of a certain amount’. Task A says ‘3 out of 5 people travel with a season ticket’. Task B says ‘Out of 800 travellers, 200 drink a cup of coffee’. Sharon started with another attempt to use the bar as a proportion table, but decided after a while to use the circle for this exercise (see Figure 6).

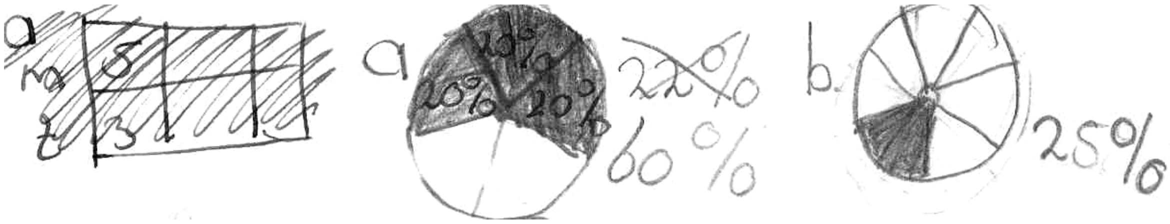

Figure 6: Sharon, providing condition

In task A Sharon draws a bar, which she makes into a proportion table. On the left-hand side she writes ‘M’ (for the Dutch word for people, mensen) and ‘T’ (for trajectkaart, a season ticket for a given train route). In the table next to ‘M’ she fills in ‘5’ and next to ‘T’ she puts a ‘3’. She does this to make it clear that 3 out of every 5 people travel with a season ticket. Apparently this model does not satisfy her, for she immediately crosses it out. She picks up a pencil instead of her pen and draws a circle. She takes a moment to think about the correct way to divide the circle into pieces, decides to divide it into 5, and colours in 2 of the 5 parts. Next to the drawing she writes her answer: ‘22 %’, though it is not clear how she has calculated this answer. After talking with her neighbour she changes it into ‘60 %’ and colours in another part of the circle, so that 3 of the 5 parts are now coloured in. Task B is next. Again she draws a circle, but this time she divides it into 8 parts. To be certain, she counts the parts again. She isn’t happy with it, so she rubs them out and draws a new circle. This time she colours in 2 of the 8 parts. Then she counts the parts that are not shaded. A period of thought follows. She seems to have difficulty calculating this answer in percentage terms. Suddenly her face brightens up and she tells her neighbour something like ‘2 out of 8 parts, that must be 25 percent’.

Designing Condition

For the next two assignments Donja decided not to use the bar she had designed earlier, but to try using figures instead. The models she designed were still context-bound, as can be seen in Figure 7. Some formalisation is visible in the transition of her model from task A to task B. Her figure drawings became more schematised, and in task B she used one figure to represent 100 people.

At first, Donja does not grasp the question: she asks the teacher how much reduction you get with a season ticket. When she reads the assignment once more, she starts to draw 5 simple figures. She gives three of them a little circle on their body and writes a ‘3’ in each little circle to represent the three out of five people who travel with a season ticket. She then stops and looks at her drawings. She calls for the teacher and tells him that she still does not understand the question. But after some thought she writes down that the 5 figures are the same as 100 % and that 1 figure then ought to be 20 %. She multiplies the 20 % by 2 as a way of telling that 40 % of travellers don't have a season ticket. Here we can see that she has calculated what percentage of the people do not have a season ticket, while the question was actually formulated the other way around.

Donja takes her ruler out of her desk for assignment B. She draws a sort of table, like one that one of her classmates used earlier. This table does not satisfy her, so she rubs it out and listens for a while to an explanation the teacher is giving to Richelle. Donja starts drawing the figures she used for the first assignment. Earlier in this lesson Maya showed the class on the blackboard how she used more formalised models of people: just a face and some dashes to represent the body. Donja adopts this model and draws 8 figures. First she places a circle around 6 figures, but then she rubs this circle out and shades the faces of the first two figures. She writes next to it: '8 P = 100 %', meaning that the 8 figures refer to 100 %. Below this she writes: '1P = 12 ½ %' and her final remark is the answer to this assignment: '25 %'.

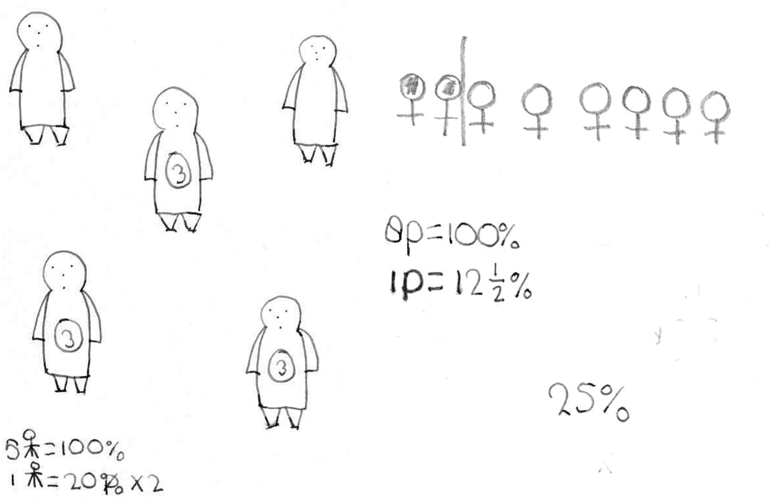

Figure 7: Donja, designing condition

Conclusion and Discussion

Research Questions

This explorative study focused on the learning processes that occurred in work on percentage tasks in real life contexts. In a follow-up research we studied the quantitative aspects of effects as pupils learn to work with either provided or designed models (Van Dijk, Van Oers, Terwel, & Van den Eeden, in press).

Clearly, we cannot draw generalising conclusions about the effects of working with provided or with designed models on the basis of developments in just the two children we focused on in this article.

For now we will merely draw some tentative conclusions, concerning the activity of model construction and use under different conditions.

Let’s return to this article's research questions, as to how pupils' learning processes proceed when models are provided by the teacher or when they are designed in co-construction. In the light of our observations the differences in pupils' handling of models were mapped out in the results paragraph .

What can we specifically say about the first research question: how do pupils' learning processes proceed when models are provided by the teacher as an aid to the solving of percentage tasks, and what can we say in particular about how pupils modelled the tasks using the models provided? On the basis of our observations of Sharon’s work, we can say that despite the teacher instruction before each task she had considerable difficulty understanding the models provided. She often applied the models blindly, as in her drawing of a bar (as explained by the teacher) with which she made a proportion table on several occasions. This provided model was not correctly understood and did not help her to solve the tasks. Only sporadically was she able to employ a bar or a circle diagram correctly, but only after several attempts.

The second research question was: how do pupils' learning processes proceed when they learn to design models, supported by each other and by the teacher, as an aid to solving percentage tasks, and what can we say in particular about how pupils modelled the tasks using co-constructively designed models? Donja showed throughout the lessons that, with the help of her group and some guidance from the teacher, she was able to think up models that she could use to solve the tasks. It must be said that the models she designed were primitive and strongly context-bound, but her development through the three lessons is visible. The bar she designed in the giraffe assignment was used again for other assignments, such as the toothpaste task. She also formalised her figure model in the season ticket task for use in the next task about people drinking coffee. Her drawings displayed a steady development, moving from a primitive drawing (see Figure 1) to a more formalised representation (see Figure 7).

Let us now go further into the matter of research question 3: the differences in how pupils dealt with models in the two conditions, ‘providing’ versus ‘designing’. We should like the reader to bear in mind the comparable starting points of the two pupils in terms of mathematical ability. While Sharon struggled to work with models she had not thought of herself, Donja did not find it difficult to apply her self-made models in solving percentage tasks. The models that Donja designed sometimes even resembled the models that were provided in the other class, for example the bars used in the giraffe and the toothpaste task. Because Donja invented them herself in co-operation with others, it was less difficult for her to apply them.

Regarding this explorative study, it seems plausible to assume that children in the upper grades of primary school are capable of designing models in co-construction. Our expectations as stated in the end of the theoretical background section of the present article are corroborated. Donja apparently was more active, goal-oriented and flexible in carrying out her tasks, in accordance to one of our expectations. Taking Donja’s maths scores into account, we can also see that no extremely high maths achievement appears to be required if pupils are to learn to design models. Still, it is important to bear in mind that these findings are only applicable in the comparison of these two girls, and can not generally be seen as true.

Limitations

In this section we will point to some limitations of our study and explore new directions for further research from two critical questions.

(1) How broad will be the bandwidth of ability in order to be able to benefit from providing or designing? Are there differential effects for high and low achieving students?

As a consequence of our restriction to medium ability pupils in our case study we cannot substantiate that low-achieving pupils equally benefit from this approach. More research into possible differential effects for high and low achieving pupils is needed. From literature and earlier studies in various domains it is known that not all students benefit equally from these kind of arrangements. Research on strategy instruction and co-operation in small groups indicates that differential effects can be expected and special instructional support is needed to foster development for high and low achieving pupils (Dar & Resh, 1986; Hattie, Biggs & Purdie, 1996; Hoek, Van den Eeden en Terwel, 1999; Mayer, 1989; Terwel, Gillies, Van den Eeden & Hoek, in press).

There are indications that less able pupils are given less opportunities to participate in small groups and are less able to adopt the perspective of their learning partners and less able to produce high quality help, and consequently gain less. Inspired by research of Dar & Resh, (1986), we believe that a certain minimum of prerequisite knowledge is needed in order to profit from a 'rich' learning environment based on 'designing'. Further research is needed to test this 'threshold' hypothesis and to test how broad the ability range might be in order to create a stimulating learning environment for all pupils.

By contrast, their high achieving counterparts seem to profit most from open, co-operative arrangements and seem to be hindered by a strict structuring prescription because they are more able to rely on own resources and strategies to design their own models. There are also indications that in the process of co-construction high ability pupils participate more and produce higher quality designs and explanations and are more able to adapt their explanations to their fellow pupils.

(2) Are the findings in this study in principle transferable to other topics in mathematics e.g. proportions, measuring, or fractions? We think that this will be the case if the program and the teacher facilitate the design of models which are full fledged in respect to future progress in mathematics. It seems a crucial point to mention that not all models designed by the pupils have equal prospects for the future development of a pupil in mathematics. It is precisely for this reason that the curriculum materials and especially the teacher should guide the pupils well in the process of 'reinvention' in order to avoid blind alleys from which a way out is blocked to more promising developments in the future. In the teaching of fractions, e.g. models like a pie or pizza, although attractive, realistic and motivating, seems to have less value in the long run as compared to the more abstract number line and its realistic derivations. It is the teacher who can guide pupils in the reinvention process of moving to and from horizontal and vertical mathematisation.

<u>Follow-up Studies</u>

In a quasi-experimental follow-up study with a pretest posttest control group design, the experience and knowledge obtained in the present study will be used for another study. For the purposes of this next study, the existing series of lessons will be expanded and carried out in ten classes. Five classes will work with the providing condition and five with the designing condition. In this follow-up research we will focus on the quantifiable learning outcomes as pupils learn to work with either provided or with designed models, focusing particularly on the issue of transfer.

The next study should make clear whether the tentative conclusions stated above also hold for other topics, pupils, groups and schools (Van Dijk, Van Oers, Terwel, & Van den Eeden, in press). The short series of lessons used in this explorative study have provided useful insights into the modelling of concrete situations and its possible effects on the learning processes of pupils. In this exploratory study we could probe the two conditions that will be the foundation for our future intervention study.

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3. Strategic Learning in Primary Mathematics Education: Effects of an Experimental Program in Modeling[5]

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Abstract

In most strategy research the focus is on ready-made models provided by the teacher or textbook. However, in this research project the effects are described of an experimental program in primary math education, concerning the construction and use of models by pupils in guided co-construction. This field experiment with an experimental group and a control group involved 238 grade-5 pupils. In a series of experimental lessons, pupils were taught to design models as a tool in the learning of percentages. The results of the experimental program were compared with the outcomes of a program in the control group, based on the teachers' strategy of 'directly providing models' to the pupils. The conclusion, then, is that children in the experimental condition significantly outperform children in the control condition.

Introduction

Over the past two decades, there has been growing interest in the use of strategies and models in primary math education (Davydov, 1988; Gravemeijer, 1994, 1997). A major issue is the way in which pupils learn to work with models in the process of acquiring knowledge in a conscious and strategic way (Taconis, 1995). Traditionally, most research projects studied how pupils learn to work with ready-made models, provided by the teacher or the textbooks (Mayer, 1989). But seen in the light of recent developments in realistic math education theories and socio-constructivist theories, pupils' contributions are increasingly considered highly important for the learning process. However, with regard to the actual educational use of strategies and models, this insight has not pervaded the mathematics classrooms yet. Therefore, this study will describe and analyse the effects of an experimental program in primary math education, concerning the use and construction of models.

Theoretical Background

Many studies have shown positive effects of the use of strategies in learning on academic achievement (Alexander, Graham & Harris, 1998; Ertmer & Newby, 1996; Hoek, Van den Eeden & Terwel, 1999; Weinstein, 1994; Weinstein, Husman, & Dierking, 2000; Weinstein & Mayer, 1986). In our research program, the 'strategic learning' notion links up primarily with the work of the Dutch educational psychologist Van Parreren and as such it is also closely connected with the activity-theoretical interpretation of human activity, development and learning (see Leont'ev, 1978; van Oers, 1990; Van Parreren, 1993). We distinguish two types of actions within strategic learning:

1. Actions that are directly performed on material or mental objects, aiming at the change of those objects in the direction of a goal (performance actions);
2. Actions that deliberately regulate these performance actions in accordance with the actor's interests, motives, plans, goals (regulatory actions).

Firstly, 'Strategic learning' implies that pupils follow a certain procedure or method on the basis of an analysis of the problem situation. Here one can think of approaches, techniques, algorithms or heuristics to solve a problem. As part of such procedures, the heuristic representation of the problem situation by the choice or design of models is considered to be an important element. According to Van

Parreren, strategic learning examples are 'learning how one solves word problems, writes an essay, understands a difficult text or throws a ball as far as possible' (Van Parreren, 1993, pp. 58, 59, 64 & 71). Characteristically, in these examples there is no unique and fixed method to be acquired in order to reach the goal. The performance always requires exploration to some extent.

Secondly, the actions mentioned under point 2 are called strategic actions, because they coordinate, organize, and regulate the performative and exploratory actions towards the goal. Hence, strategic actions can also be conceived of as a kind of meta-actions. Strategic learning, then, is defined as learning on the basis of such regulatory actions, or as learning these strategic actions themselves. To put it more concisely, one could say that strategic learning amounts to an activity-theoretical version of reflective learning or meta-cognitive learning.

Van Parreren states that both kinds of actions are not to be disconnected. It is precisely for this reason that in our research program strategic learning is conceived as embedded in the curriculum and as connected to domain specific knowledge and concepts.

In this approach to learning, the knowledge acquisition process is seen as a constructive process, based on the production of new symbolic constructions that acquire actional meaning (van Oers, 1996, 2000) for the deliberate regulation of goal-directed actions. The meaning of the new symbolic constructions is basically derived from a social process of meaning negotiation.

Sociocultural theorists argue that meaningful knowledge construction is only possible if pupils have the opportunity to work together in a cultural practice and learn from each other in that context. In this view, collaborative learning under teacher supervision is seen as a basic pattern for the organization of learning processes. The joint activity can be conceived as a kind of 'distributed cognition' (Cole & Engeström, 1993), in which each participant can profit from cultural resources which are offered by other participants and by materials used in the activity. These resources enable each participant to accomplish more than they could do on their own. In this way, participating in such activity can be seen as jointly constructing a zone of proximal development (see Moll & Whitmore, 1993; Van Oers, 1995). In the higher grades of primary education the joint activity is increasingly becoming a discipline-based learning activity (Davydov, 1988), in which pupils are stimulated to develop and compare their own problem solving methods. The negotiation of possible approaches and ways of problem solving will then contribute to the development of a solution supported by all participants (Cobb, Wood & Yackel, 1993; Inagaki, Hatano & Morita, 1998).

Aspects of the sociocultural perspective, as described above, can also be found in the instructional design theory of realistic mathematics education in the Netherlands (Gravemeijer & Terwel, 2000). Following Freudenthal (1991), we assume that guided re-invention of mathematics by pupils in a community of learners plays a mayor role. How does this active knowledge construction in a 'community of learners' take place? Research into mathematics education has led to the development of an approach in which the learning of mathematics is conceived as construction of meaning and understanding. This meaning formation is, however, impossible without the construction of new tools for communication about those meanings, i.e. it needs the construction of new symbols (see van Oers, 1996, 2000). In mathematics education, this often implies the construction of symbolic tools, based on the modelling of reality (De Corte, Greer & Verschaffel, 1996). Together with the idea of math as both an active and a constructive process, the social character of this process is underlined. The pupils are not to be seen as isolated information processors (see van Oers, 1990), but as participants in the class as a (mathematical) community (Cobb et al, 1993).

In realistic mathematics education two strategies are used in order to challenge children to participate in this reconstruction process. First, bringing pupils into situations that make sense to them and provide them with opportunities to experience how mathematics was developed in cultural history. Second, encouraging pupils to produce spontaneous, informal, self-invented problem solutions (Gravemeijer 1994). Pupils' informal strategies can often be interpreted as anticipating more formal procedures. Mathematizing such solution procedures creates opportunities for a reinvention process (Gravemeijer, 1994).

Nowadays, many researchers in the mathematics community are inspired by socio-constructivism (Cobb et al, 1993; Cobb and Bowers, 1999; Gravemeijer, 1997). Socio-constructivists argue that all knowledge is self-constructed in an interactive process with others. Hence, in mathematics lessons there must be a climate of open dispute, encouraging pupils to generate solutions and answers that make sense to them. In such a climate pupils must pool their self-invented solutions and interpretations in order to create a common sense that constitutes the basis for their community, for their intellectual exchanges, and for their negotiations of meaningful solution(s) to the problem(s) at hand (see Edwards & Mercer, 1987). In this process, mt is the teacher's task to stimulate the pupils' acculturation process into the wider mathematical community and by doing so to support the pupils' understandings (Cobb et al, 1993).

Within the socio-constructivist approach, mathematical tasks are often embedded in day-to-day contexts. In schools, pupils do not always have access to skills needed to solve context-bound tasks. They experience problems with the application of newly learned actions to new situations, although they seem to master the basic actions. This problem of the strategy application in mathematical problems in day-to-day contexts is frequently mentioned in the literature (Säljö and Greer, 1997). We interpret this phenomenon as a consequence of the lack of strategic learning abilities, particularly a lack of the construction and use of models for the regulation of problem solving actions. Several strategies needed for the solving of context-bound tasks are discussed in the literature (Rosenshine, Meister & Chapman, 1996). A possible solution to this problem might be a training program in social and cognitive strategies, in which designing and modelling would be crucial (Hattie, Biggs & Purdie, 1996; Hoek, Terwel & Van den Eeden, 1997; Hoek et al, 1999).

Modelling is seen as a powerful method for the structuring of open and ill-defined problems. Models can be seen as a tool, for example for building a bridge to overcome the gap from concrete situations to abstract mathematics. As such, a model is a tool for strategic thinking. It structures thinking: it is a symbol that refers to cultural historical knowledge and guides thinking to possibly successful solutions (Davydov, 1990). A symbol can be used for communication and discourse. However, a problem commonly found in the use of models that are developed by teachers or designers of methods is that pupils do not always grasp immediately what adults see in it. This is the case when models do not properly fit in with the pupils' own way of structuring a problem situation. In this case, a model should support the pupils' own exploratory actions, instead of transmitting the adult's way of performance.

In our view, modelling can be embodied in a variety of ways: it can take the form of making a drawing, a diagram, a formal-symbolic representation, or a story. In any case, it implies that an object (a situation, an action, a thing) is represented in a simplified way. The model representation articulates the structural aspects of the object involved, and obviously it holds the pretension of replacing that object for the time being. In self-invented models, the subject thus expresses his or her view on the object involved and as such also expresses his or her informal appraisal of the situation. This informal model is a stepping-stone for bridging the gap between informal (personal) representations and formal, mathematical representations.

It is assumed that models are useful for learning mathematics (Gravemeijer, 1994; 1997; Mayer, 1989; 1996; 1999). It is, however, doubtful whether the imposition of ready-made models on pupils' thinking, will help them master the mathematizing activity. Following Freudenthal (1991), we assume that the appropriation of the mathematical structuring ability might be more helpful than the mastery of mathematical structures. To put it differently: we advocate math as a human activity rather than math as a ready-made system. Therefore we wondered if it might be more rewarding if we taught pupils to design models themselves, in co-construction with peers and the teacher (Gravemeijer & Terwel, 2000). The idea behind this point of view was that pupils who learn to design models in co-construction would choose to make informal models that are more in harmony with their competence level and their day-to-day understandings. Therefore, we suppose that it should help them to better understand the subject matter the model is about. Having learned how to construct adequate and more formal models presumably places the pupils in a better position for solving new problems for which no ready-made models are provided or available.

The optimal way for the appropriation of the modelling activity in the context of mathematical problem solving is still a matter of debate. In our research we want to shed some light on this process and its value for the development of mathematical thinking.

In the process of problem solving, pupils initially construct informal models, which are commented upon by their peers or teachers. As a result the pupils are stimulated to modify their models and improve them in order to bring them more in accordance with the consensual view or the requirements of the situation (Van Dijk, Van Oers & Terwel, in press). By doing so, as Gravemeijer (1994; 1997) has pointed out, the pupils gradually reconstruct their model of a situation into a model for the situation. We theorized that by doing so pupils will gradually appropriate the mathematizing process (i.e. the activity of mathematical structuring) more profoundly than by adopting ready-made mathematical structures and performance actions.

Research Questions and Hypothesis

To test the above hypothesis a study was conducted in which pupils were taught to work with models according to the principle of guided co-construction. This meant they learned to design models as a tool in the learning of mathematical actions. As mathematical subject matter we chose percentages and graphs. In our view, pupils should be seen as co-designers of models. Co-designing may lead to an upgrade in their level of thinking, caused by reflection and discussion during the co-construction of knowledge. The teacher essentially takes a crucial role in this process as one of the participants in the activity. The teacher guides the construction process by asking critical questions, making objections, providing additional information or even suggesting hypothetical solutions when the pupils' solution process fails. In brief: the teacher guides the strategic (reflective) learning process and promotes the construction and reconstruction of models.

The study's research question is: What are the effects of acquiring models by co-constructive learning as compared to the mastery of models by an expository teaching approach? This co-constructive learning approach can be characterized as a form of activity in which the pupils participate as model designers in a mathematical context, and jointly construct mathematical models for the solution of complex problems.

Our hypothesis is that this teaching method will have positive effects, such as changes in learning processes and better learning results of pupils in mathematics. The rationale behind this hypothesis: pupils will gain deeper insight into math problems structures because they will have gained experience in designing and will have been involved in strategic learning processes, in which exploration, reflection and negotiation play crucial roles. We expect this deepened insight to help pupils see through structures of new and unfamiliar problems. The strategy acquired can be used to solve new problems.

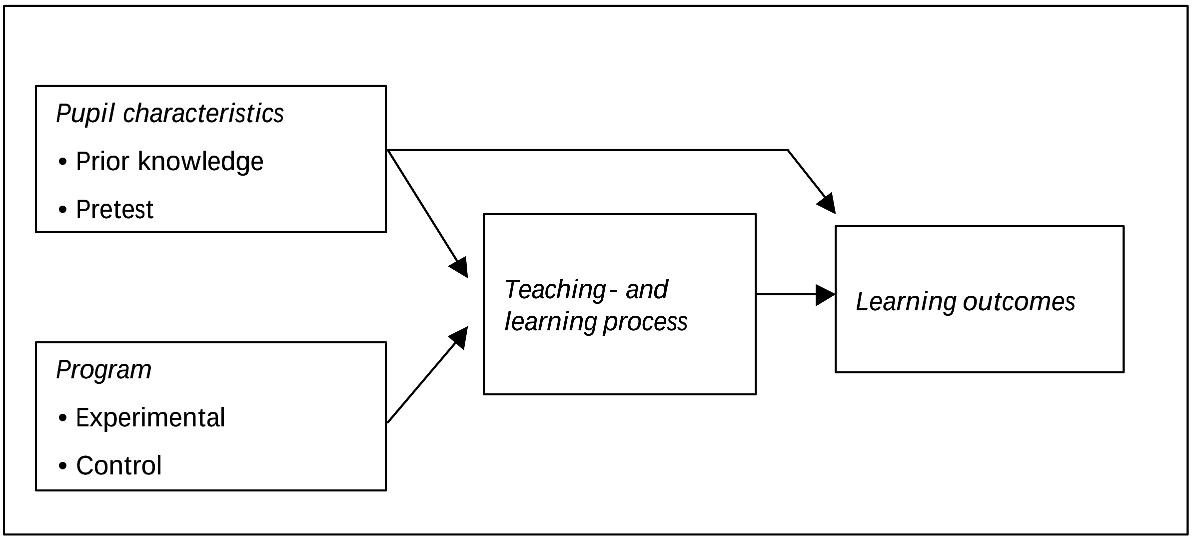

Figure 1: Main variables and their relationships.

The main variables and their relationships in the study are represented in Figure 1. In this article we will focus in particular on the influence of the intervention and pupil characteristics on the learning outcomes.

Pupil characteristics are measured by a pretest. Learning outcomes are determined by a posttest that measures the knowledge pupils have acquired afterwards. The teaching and learning processes were described in protocols on the basis of direct observations and video recordings. Program refers to the intervention, which will be described in more detail in the curriculum materials section. The program consists of 13 lessons.

Methods

Research Design and Participants

In this field experiment, with an experimental group and a control group, 8 schools, 10 classes, 10 teachers and 238 grade-5 pupils (age 10-11 years) were involved. A pretest-posttest control group design was used. 117 pupils were assigned to the 'providing' condition: teacher-made models were provided to the pupils while they were learning the percentage-concept. This 'model providing' approach is the way in which regular education takes place in most primary schools in the Netherlands. The experimental ('co-constructing') group, consisting of 121 pupils, was exposed to the same mathematical content, but here the emphasis was on 'guided co-construction' of models by pupils and teacher.

The schools participating in the experiment were situated in, or near two cities in the centre of the Netherlands. Experimental and control schools were either middle-class schools or schools with high proportions of ethnic minority group children. Both types of schools were equally represented in the two conditions.

The grade-5 classes were randomly assigned to the control condition or the experimental condition. At the schools that participated with two teachers and two classes, one class was assigned to the control condition, and the other class was assigned to the experimental condition. There were no 'drop-outs' (teachers or classes) during the study.

Curriculum Materials

For the purpose of this study a series of lessons was developed on the subject of percentages and graphs. Given our theoretical assumptions, this purpose, however, cannot be achieved with any curriculum. As Gravemeijer stated: 'a teaching strategy that leads to comparing and explaining solutions by pupils is only possible if the learning sequence consists of contextual problems that give rise to a variety of solution procedures. It is the variety that allows for discussions about adequacy and efficiency, which in turn leads to a reflection on these procedures from a mathematical point of view' (1994, p. 90).

The tasks and assignments to be done in the lessons were open, complex problems. They were chosen from current math methods used in the Netherlands (Pluspunt, Wereld in Getallen Nieuw) and in the United States (Mathematics in Context). Furthermore, exercises were used which had been developed for MILE (Multimedia Interactive Learning Environment), a project of the Freudenthal Institute for teacher trainees. All the exercises used were adapted to fit the purposes of the intervention intended.

In addition, a teachers' manual was developed containing the main ideas and principles of the approaches.

The intervention consisted of a one-hour lesson every day for (almost) three weeks. It consisted of 13 lessons: one lesson in which the children learned about strategy use, models and their functions, and 12 lessons on percentages and graphs. Four lessons dealt with percentages, another four lessons dealt with graphs. In the rest of the lessons, percentages and graphs were combined. To improve the readability and save space in this article, we have chosen to show examples of the percentages lessons only, although tasks of graphs have taken an equal amount of time in the lessons and the tests. Of course, the posttest also contained both topics.

In both conditions, the same amount of time was spent on the tasks. The teachers limited their lessons to one hour a day, not only to keep the conditions alike, but also to keep the attention and concentration of the children as high as possible. The mathematical content in both conditions was the same, but the method (providing or designing) differed, as will be elaborated in the curriculum materials paragraph.

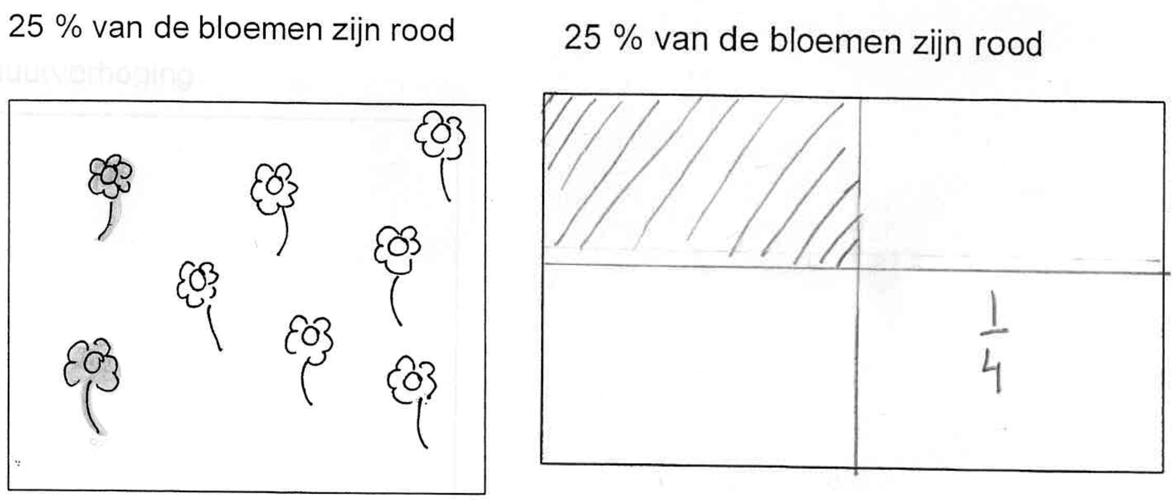

Figure 2: Examples of a task in the two conditions. First the results of the task in the providing condition in which the model (here: 8 flowers) is ready-made.Second a task in the designing condition: the model was invented by one of the pupils. The text above the assignment reads: '25 % of the flowers are red'.

For each condition a particular version of the program was made. These versions show a number of essential differences. Firstly, in the way pupils learned to work with models: in the experimental condition the pupils co-constructively were encouraged to design their own models, as a tool for the learning of percentages and graphs. In the control condition the pupils learned to apply ready-made models provided by the teacher. They did not learn to design or choose models themselves. Secondly, the children in the experimental condition were asked to work in pairs and to discuss the models they had designed, before the models were discussed in class. The children in the control condition worked individually on the tasks.

The following is a typical example of the kind of task that was assigned in both conditions. In some assignments pupils were asked to recognize percentages and shade this amount on a given model (provided condition) or invent a representation (designing condition) of this amount. An example can be found in Figure 2.

Other tasks, such as the coffee pot assignments, were more open-ended and ill structured (see Figure 3). In this task the teacher told a story about a luxury cinema, where during the movie one could ask for coffee by pulling a button. The images in Figure 3 show a seating plan in the cinema and a coffeepot that is filled for three-quarters.

6. Genoeg koffie?

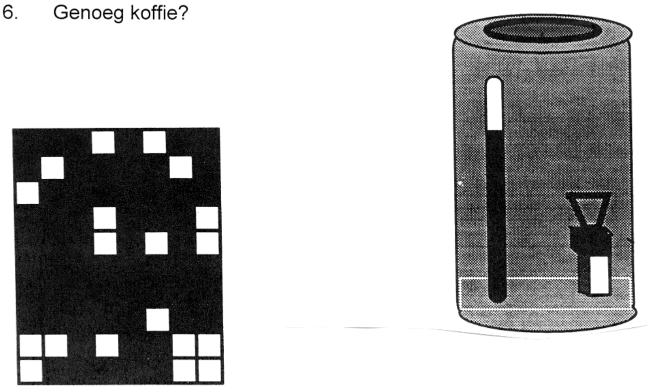

Figure 3: The introduction of the coffeepot assignment in both conditions. The text above the task means: 'Will there be enough coffee?'

The question is whether there is enough left to provide everyone who pressed the button with one cup of coffee. The pupils have several decisions to make: what do the black and white blocks mean on the seating plan? How many cups of coffee does the pot contain? Most pupils decided that the black blocks stood for people who wanted coffee and that a whole coffeepot should contain enough cups to serve everyone in the cinema once: that would be 80 cups, as can be seen on the seating plan. About three quarters of the pot is filled, so about 60 cups of coffee are available. This is not enough coffee for the 61 coffee-drinking people. In this task, the relation between fractions and percentages was explored and explained when necessary. See Figure 4 for examples of the models the children designed, or were provided with, in the cinema context.

Indeed, one could argue that it is possible to solve these problems with already existing knowledge of fractions only, and without any use of model construction. Our point of view is that percentages and fractions are conceptually interwoven, and that it is very difficult (if not impossible) to learn about percentages without some earlier knowledge of fractions. Therefore, in the teacher's manual we emphasized the need to explicitly discuss the connections between fractions and percentages. We expect pupils in the experimental condition to be better able to master the conceptual basis of percentages and graphs, due to their work at modelling.

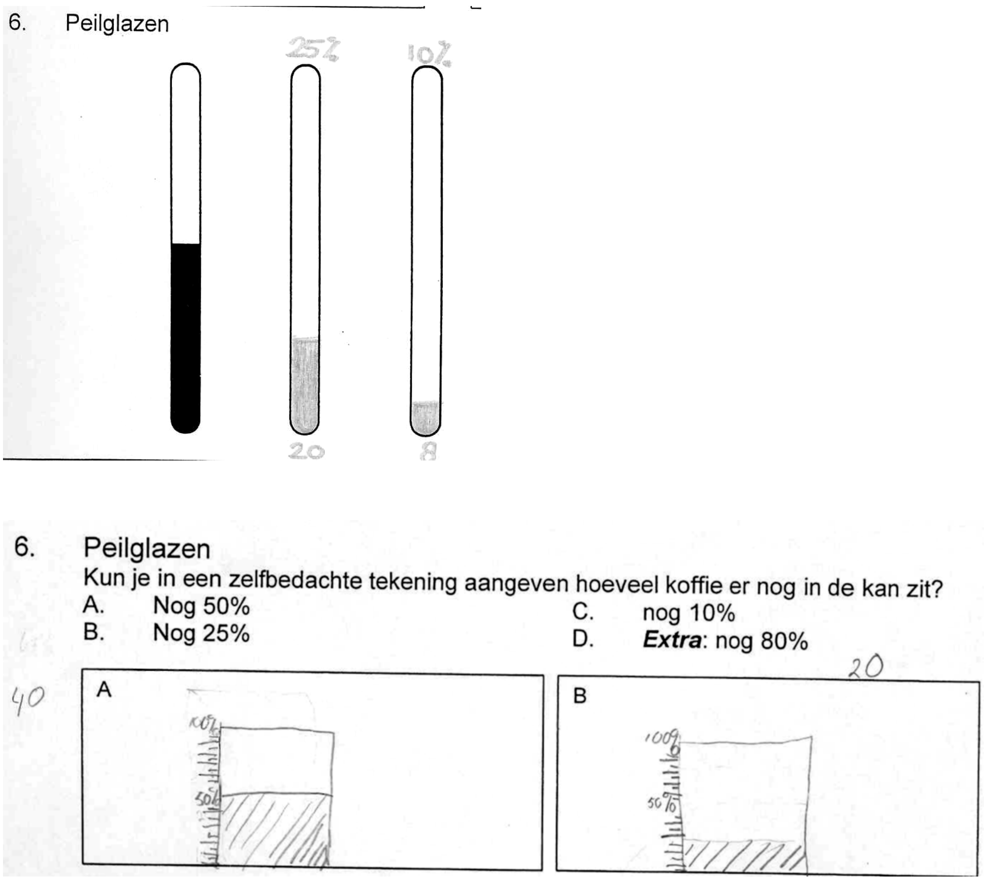

Figure 4: Differences in model use. First an example of a task in the providing condition in which the model is provided and second a task in the designing condition in which the pupils invent representations themselves. The text above the assignment reads: 'can you make a drawing, which shows how much coffee is left in the pot? A. 50 % coffee left B. 25 % coffee left'.

We suppose that their learning process of 'how to model a situation' leads to a deeper insight, and more flexibility. As a consequence, this leads to better results on the posttest for the experimental group, even when they only use their knowledge of fractions. Considering the difficulty level of the problems in the test, we expect that the solution of these problems would have been impossible, merely on the basis of rote knowledge of fractions. Therefore, we assume that a higher score of the experimental group is due to an improvement of insight in model making, and consequently of a better use of fractions and percentages.

<u>Classroom Activities</u>

We will now further elaborate on the kinds of classroom activities that occurred in both conditions. We will start with a description of the classroom activities in the providing condition, afterwards we will continue with the designing condition. The coffee pot assignment, already discussed in the last paragraph, will be taken as an example.

After the introduction of this assignment the pupils of the providing condition worked with a ready-made model of the pot with various coffee amounts left. They were asked to explain in percentages (50%, 25%, 10% and 80%) how much coffee remained and how many cups of coffee were left in the pot. The teacher walked around and gave help to pupils who needed it. Their model use and solutions were then discussed in a class discussion. A protocol is given of the last exercise (80% of the coffeepot is filled), which shows how teacher and pupils worked on the assignment.

Teacher:	The last task was the most difficult one. Danny, what do you think? How many cups of coffee does the pot contain when it is filled for 80 %?
Danny:	One cup..
Kids:	Huh?
Teacher:	One cup? So if you serve one cup of coffee, your pot will be empty? (Pointing to the model with 80 % on it) But we saw in the first model that when the coffeepot contains 50 % coffee, we have coffee left for 40 people. Let's help Danny. Fenna, what do you think?
Fenna:	64 cups.
Teacher:	64. Fenna, we would really like to know how you found this amount.
Fenna:	Well, if you know that 50 % is half, you just do plus 8, plus 8 and plus 8.
Kids:	How? I don't understand! I don't get it!
Teacher:	Wait, I do understand what she's saying! Look at this (points at an earlier model): 50 % was the same as 40 cups. And we also calculated what was 10 %, that was 8 cups. So Fenna did: 50 %, plus 3 times 10 %, which is 80 %. So she did: 40 cups, plus 3 times 8 cups, is 64 cups of coffee. Any one who did it another way?
Ben:	I did 8 cups, which is 10 %, 8 times.
Teacher:	Very well!

In contrast, the pupils in the designing condition were asked to develop a drawing in which one could see that the pot contained 50% (or 25%, 10% and 80%) of the total coffee amount. The teachers' way of working let the pupils themselves think about the problem solution first, without giving them ready-made solutions. They were stimulated to visualize and represent the situation in a model. No ready-made model was provided and it should be noted that teachers in the designing condition did not ask the pupils to use a certain model. While the pupils were designing their models, the teacher walked around the classroom. The teacher helped where necessary, and asked critical questions to individual children about the applicability of the designed models. The pupils were stimulated to discuss their models with their neighbours, and to make improvements. During this process, the teacher asked several pupils to draw their model on the blackboard. When most pupils had at least thought of a way to represent the problem, the teacher started a class discussion. The researchers expected that children who saw the advantages of one of the models discussed, would eventually adopt it on their own initiative. A protocol of the designing condition is given below. It starts at the point where several models on the blackboard are discussed. Each of these models was meant to represent the coffee pot, which was for 25% filled with coffee.

Teacher:	Bart, first we'll take a look at your model. Can you explain what you did?
Bart:	Yes, I drew a coffeepot. (It's a coffeepot in a concrete way, complete with handle and other accessories)
Teacher:	OK, and how can we see it is for 25 % filled with coffee?
Bart:	(points to his model) This little line, at half of the pot.
Teacher:	Aha, but you also made a line at 50 % and 75 %. How can I see that you mean it is for 25% filled?
Bart:	Oh, I forgot! (Quickly shades at the blackboard the lowest quarter of the coffeepot)
Teacher:	That's better. Ann, tell us about your ideas?
Ann:	I made a sort of thermometer, and I put 100 small lines in it. And then I coloured the first 25 lines red.
Teacher:	OK, and how did you figure out how many cups of coffee this pot contains?
Ann:	I knew that 25 out of 100 is a quarter.
Teacher:	Can you show it on your model?
Ann:	(Points at her model)
Teacher:	And how did that help you?
Ann:	Well, I knew the total pot contained 80 cups, and then I took a quarter out of 80.
Teacher:	Good. Did it take you long to draw this model?
Ann:	Uh... yes, quite long…
Teacher:	Who can think of a solution to make this model easier to draw?
Jesse:	I would not draw all those 100 lines, but only the most important ones, like 0%, and 100%.
Teacher:	OK, that would make it much easier. Linda, can you explain your model?
Linda:	I just made a kind of a bar. That's supposed to be the coffeepot. And then I shaded the first quarter of it.
Teacher:	Why did you choose a bar to represent the coffeepot? It doesn't look like a coffeepot at all!
Linda:	Because the model doesn't really have to look like a coffeepot. I think a bar like this is just easy to draw! And you can use it in every situation.
Teacher:	Well, we saw three models. Who made a model like Bart's? (Several children raise their hand) And who has made a model more like Linda's? (A couple of children raise their hand). Jim, you didn't raise your hand. What model do you like best, Bart's, Ann's or Linda's?
Jim:	Linda's ..
Teacher:	Why?
Jim:	Because it's fast, you don't need to draw the whole coffeepot. And you don't need to draw 100 lines, like in Ann's model. Linda's is simpler.
Teacher:	These models on the blackboard are all OK, but they do differ at some points. Some models are easy to use in this situation, but also in other situations. And we agreed earlier that a model has to be as simple as possible, without all kinds of extras. And that it is important

> that you can draw it quickly. That's why Linda's model is a good one. You can use it also in other situations, even when it is not about coffeepots, it is simple and it is quick.

In the providing group, as could be seen in the first protocol, the teacher stimulated the children to refer to the model provided while explaining their thinking. They gave their explanation in words and the teacher pointed at the model on the blackboard. They were expected to apply the model provided in their solutions of the exercise.

As the protocol of the designing group shows, some of the pupils in the designing condition were given a moment to explain their model and their thinking to the other children. The others could suggest improvements, and the teacher asked critical questions to stimulate reflection and to provoke improvements. The whole group learned about the process of model designing, and how to use models to solve math problems. In short, the difference in working with models in both conditions is that in the providing condition children are asked to use models that already exist without changing these models to their own needs, while in the designing condition the children use models they re-invent themselves and therefore can change models they like for use in other situations.

<u>Procedures</u>

The experiment started with teacher interviews, by which the researcher could gain insight into aspects such as teacher attitudes and perspectives on mathematics, collaborative learning and other important characteristics of the individual teachers. Then, for each of the conditions separately, a workshop was organized in which the program and the teacher manual were explained and materials were discussed. The teachers participating in the experiment joined one of the two workshops. In the autumn of 1999 all teachers started the program in the same week and ended the program three weeks later.

All teachers were visited at least two times in the course of the investigation, to give them opportunities to ask questions about the material and to give the researcher the opportunity to control the course of the lessons, the validity of the conditions and the integrity of the intervention. In addition, the teachers filled out a small questionnaire after each lesson. At the end of the program a second interview was held. This interview would give the researcher an impression of the teachers' evaluation of the program implemented.

<u>Instruments and Analyses</u>

A broad spectrum of methods and instruments was used to describe and measure the teaching - learning processes and outcomes. We made direct, participant observations, made audio and video registrations, administered teacher and pupil questionnaires, a standardized pretest and a curriculum specific posttest. In this article the analysis is restricted to the pretest and the posttest. Just as examples, a few observation samples of the teaching – learning processes were added.

We administered a standardized pretest (containing 150 items) and a curriculum specific posttest, (containing 27 items) in order to find effects of the program in terms of learning results. The tests proved to be reliable with alphas ranging respectively .90 and .83. It should be noted that the alpha of the standardized pretest is based on oral reports of the institute that developed the test and that no publications are available yet.

The posttest measured the pupils' achievement regarding percentages and graphs in a quantitative way. The answers were assessed as either 'correct' or 'false'.

Results

The data presented in this paper were analysed with the statistical program SPSS, and MLwiN. In order to determine the intervention effects, one-way ANOVA, regression analysis and effect sizes were used. In Table 1 the characteristics of the pretest and posttest distribution are presented.

Table 1: *Characteristics of the Distributions of the Standardized Pretest and the Curriculum Specific Posttest for All Pupils (N-Pupils = 238, N-classes = 10).*

	Mean	S.D.	Min	Max
Control program N-pupils = 117				
N-classes = 5				
Pretest	136.07	22.02	49	189
Posttest	43.55	14.76	10	73
Experimental program N-pupils = 121				
N-classes = 5				
Pretest	137.87	17.80	95	181
Posttest	49.40	13.76	17	75

From Table 1 it can be seen that the experimental group gains more on the posttest than the control group. In a one-way analysis of variance no significant differences on the pretest scores between the two groups were found (pretest: $F(1,237) = .484$, $p = .487$). The difference in posttest score is 5.85 in favour of the experimental group. A one-way analysis of variance resulted in significant differences between the posttest scores ($F(1, 243) = 8.203$, $p = .005$). The analysis of covariance with the pretest as covariate also shows significant differences on the posttest ($F(2,235) = 208,256$, $p = .000$). Hence, it can be concluded that in general there is a positive effect of the experimental program on learning results. This result is in line with the hypothesis of the study. No interactions were found, therefore it was allowed to proceed with regression analysis.

We considered the effects of the variable 'intervention' and 'pretest' on the learning outcomes. A multiple linear regression analysis was conducted, in which a dummy variable was created for intervention (0 stands for the control group [providing condition] and 1 for the experimental group [designing condition]). The variables 'pretest' and 'intervention' were added with the stepwise method. The outcomes are presented in Table 2.

Table 2: *Summary of Multiple Linear Regression Analysis for Variables Predicting the Scores on Posttest (N=238)*

Variable	B	SE B	ß
Step 1			
Pretest	.569	.030	.781*
Step 2			
Pretest	.563	.029	.774*
Intervention	4.927	1.137	.170*

Note. R^2 = . 61 for Step 1; R^2 = . 639 for Step 2;
R^2 change = .029 for Step 2; (ps < .05).
* p < .000.

The data distributions were checked for the assumptions of linearity, heteroscedasticity, and outliers. From Table 2 it can be concluded that the variable pretest already explains 61 % of the posttest variance. The variable intervention contributes with another 3 %. We decided to compare the regression equations of the two groups in this study. In Figure 5 we plotted the individual results of the children in the providing group and the experimental group on the pretest and posttest. For these tests we calculated the regression lines and the prediction intervals for single observations (confidence level = 95%). The regression equation of the control group is: posttest = – 31.027 + .547 pretest. The regression equation of the experimental group is: posttest = – 31.562 + .587 pretest. This means that all children profit from being in the experimental group.

Effect sizes were calculated: this is the difference between the posttest means of the intervention group and that of the control group, divided by the standard deviation of the control group. The effect size in this study is .40. This is considered as a relatively small effect (Cohen, 1988). Effect sizes of +.20 - .25 are of practical significance in educational environments (Slavin, 1996, p. 31). An effect size of .40 means that if an average pupil (50th percentile) of the control group had been in the experimental group, this pupil now would have scored on the 66th percentile of the control group (from .50 to .66), which is a relevant gain (for an explanation of this computation, see Rosenshine et al, 1996).

In addition of the standard covariance analysis, which showed no interaction effects, a multilevel analysis was done, since it explicitly views that interaction as an effect of Condition on class level on the influence of the Pretest on the Posttest . The differences between the conditions could be due to a few classes in the experimental or control group. Although the number of classes is limited for this type of analysis, according to Snijders & Bosker (1999) this analysis is justified by N =10 at class level and n =100 at pupil level. Our sample size is N =10 at class level and n =238 at pupil level.

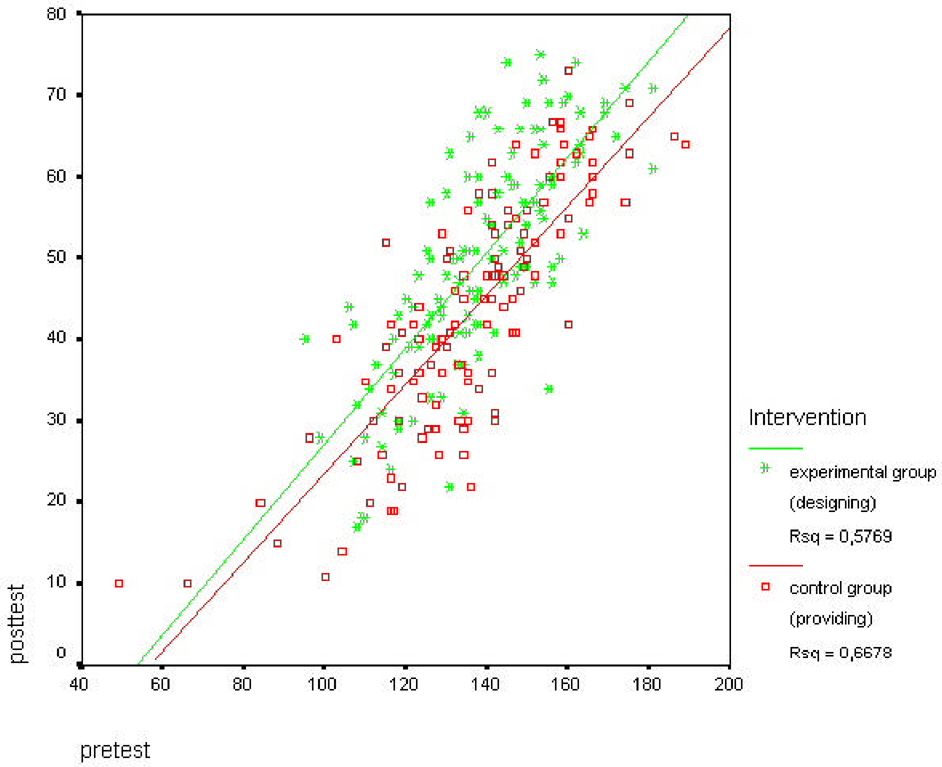

Figure 5: Comparison of experimental group and control group

Before presenting the outcomes of the multilevel analysis some preliminary remarks have to be made about the meaning of the intercept- and the slope coefficients that are shown in Table 3. In Model 1, the variance in the dependent variable (Posttest) is divided in a component at pupil level (173.70 or 84 percent) and a part at class level (33.48 or 16 percent).

In model 2, three variables are introduced: Pretest at pupil level, and Mean Pretest and Condition both at class level. The coefficients in the fixed part can be viewed as the conventional un-standardised regression coefficients. For example, in Table 3, the coefficient .54 means that a change of one unit on the Pretest (pre-knowledge in Mathematics) scale will result in a change of .54 unit on the Posttest (Achievement in Mathematics) scale. The descriptives from Table 1 (means) can be used to estimate the relative magnitude of the effects for an average pupil by multiplying the coefficient by the corresponding mean for the Pretest.

Let us also give an example of a class level variable from Table 3 by referring to the coefficient 4.66. Pupils in the experimental condition get a learning gain ('bonus') of 4.66 scale points on their Posttest score.

Table 3: *Results of the Multilevel Analysis, regarding the Posttest in Mathematics as the Dependent Variable.*

	Model 1	Model 2
Fixed part		
Pupil level		
Pretest		.54 (0.03)
Class level		
Mean pretest		- -
Condition		4.66 (1.69)
Random part		
Pupil level		
Variance	173.70 (16.31)	69.89 (6.56)
Class level		
Variance	33.48 (18.21)	4.04 (3.15)
-2 * log (lh)	1911.97	1687.67

Note. *p = .05, Standard error between parenthesis, N-Pupils = 238, N-classes = 10.*
'-' = Non-significant effects.

The coefficients in the random part of Table 3 refer to the variances of disturbance terms that are left after introduction of the pupil- and class variables in the analysis. To put it differently, the random part concerns the residual (unexplained) variance after having introduced all independent variables in the analysis. The variance left at pupil level is called within class residual variance; the variance left at class level is called between class residual variance. The outcomes of the multilevel analysis are presented in Table 3.

In the multilevel analysis it turned out that the slopes between the classes did not differ significantly. Hence, we decided to look further for differences in intercepts between the classes to be explained by differences in class mean on the Pretest (Mean Pretest). The result of this analysis also shows no effects. The final conclusion is that the outcome of the multilevel analysis clearly confirms the outcome of the earlier analyses at the individual level (regression analysis and analyse of covariance). No interaction effects and no class effects could be found. The introduced variables in model 2 explained a large part of the variance on both levels. The non-significant contribution of the class variable Mean Pretest was unexpected. In several studies was found that the mean class pretest score contributes to the scores at the Posttest (see for example Van den Eeden & Terwel, 1994). However, these studies concern secondary education, which means streamed classes. In primary education classes are not streamed although differences between schools can be large. Therefore, further research with larger samples is needed. The results of Table 3 are presented in a graph in Figure 6.

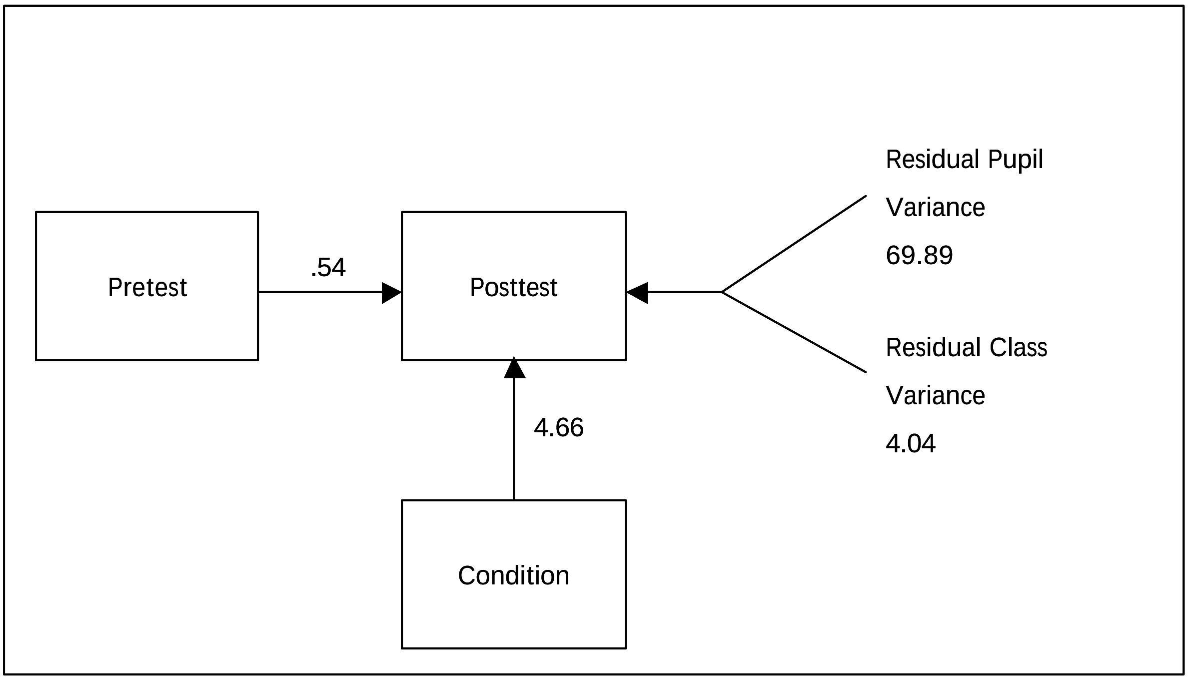

Figure 6: Diagram of the Multilevel Analysis

We hypothesized that the pupils' learning processes in the experimental group would progress in such a way that a greater increase in results on the posttest would be demonstrated, as compared to the growth of control group pupils. The results confirm our hypothesis: pupils in the experimental group significantly outperformed pupils in the control group on the posttest. Taking into account the ANOVA, regression analysis and multilevel analysis, we conclude that the experimental intervention had a significant, positive effect on the learning outcomes. Experimental pupils clearly show more progress in the learning of mathematics as compared to pupils in the control condition.

Conclusion and Discussion

This study focused on the effects of an experimental method in which children learned to design models and to draw graphs in co-construction, as a tool in working with percentages. In this article we addressed the research question: What are the effects of acquiring models by co-constructive learning as compared to the mastery of models by an expository teaching approach? We compared learning outcomes of pupils who had learned to work with percentages according to the method of 'designing models', to learning outcomes of pupils who had learned to work with percentages in a more regular way, with the teacher providing ready-made models for them.

In the course of this article we hypothesized that strategic learning, of which learning 'how to model mathematical problems' is an example, would be beneficial for the solution of mathematical problems. We expected this strategy to have positive effects, such as changes in the pupils' classroom activities, changes in learning processes, and better results of pupils on cognitive tasks. Pupils were expected to gain deeper insight into the math problem structure because of their experience in designing and their involvement in strategic learning processes.

Elsewhere (van Dijk et al, in press), we already reported an in-depth observation study on two teaching conditions on the pupils' classroom activities. In the present article we focused primarily on the effects of the conditions on the learning outcomes. The results clearly show the positive effects of the experimental program. In addition, we also had clear indications that these outcomes occurred on the basis of differences in classroom activities between the two conditions. Pupils who learned to construct models in the experimental program scored significantly better on the posttest than children who learned to work with models provided by the teacher.

How can these effects be explained? Although the outcomes of the study are in line with the expectations as elaborated in the theoretical background and the hypothesis, how can we substantiate that the intended teaching - learning activities really occurred in the classroom? Do we have empirical data about the implementation of the program, in this case the classroom activities? One limitation of this study is that the available resources for this project did not allow for systematic observation in all classes and all lessons. Causal inferences in the strict sense cannot be made because of the fact that we do not have quantitative observation measures for each of the ten classes to introduce in the equation for e.g. the regression analyses. Nevertheless, we have clear indications that the intended curriculum was realized in daily classroom practices and that the expected outcomes occurred as a consequence of these classroom activities.

In trying to make the intervention a success we put a lot of effort in the preparation and planning. In preparation of the present study, a small scale, qualitative case study was carried out. This case study – in which a 'providing' and a 'designing' classroom were contrasted, and in which two pupils were followed closely in their learning process - clearly showed the intended activities of teachers and pupils. In addition, some indications were found that the expected results of learning occurred. These in-depth descriptions and analyses of modelling processes were already reported elsewhere (Van Dijk et al, in press). From this qualitative study we received a firm foundation for the present study and a convincing case that the intended processes can be realized in normal classroom settings.

As part of the present study, which is the main study of the project, curriculum materials and a teacher manual were developed, tested and revised. Teachers received a training and in turn teachers prepared and guided their pupils in the co-construction processes of problem solving and designing models. During the experiment the researcher consulted each teacher in order to get insight in what pupils and teachers were doing in the classroom. All teachers were interviewed before and after the experiment. In addition, qualitative observations were carried out and video recordings were made.

In the 'classroom activities' section of the present article, we included a few protocol samples of these observations and video records. These fragments clearly show the differences between the two conditions, in terms of the role of the teacher and the role of the pupils. The teachers in the providing condition guided the way in which ready-made models were applied, while the teachers in the designing condition scaffold the process of model designing. At this moment the whole qualitative data set is still in the process of being analysed and cannot be incorporated in full within the scope and limits of this article.

Although we have to admit that no decisive, quantitative proof can be given, we believe that there is convincing evidence from qualitative data that the intervention really occurred in the classrooms. The outcomes of the case study, the side measures which were taken to reassure an implementation according to the intentions, the classroom consultations, the interviews with the teachers, along with the samples (protocols) of the teaching and learning processes may substantiate our claim. Moreover, it is noteworthy to remind that our hypothesis was based on a firm body of theoretical knowledge and that there are some empirical findings from literature, which point, in the same direction.

All in all, looking at the outcomes of the study and taking into consideration the strengths and limitations of our design and measurements, we tenaciously conclude that our hypothesis is confirmed. Pupils in the experimental condition outperformed their counterparts in the control group. Learning how to design models in the context of a strategy for solving mathematical problems in daily life situations, put pupils in a better position as compared to pupils who were provided with ready-made models. Our theoretical point of departure and the results of our experiment make us believe that designing models in co-construction may lead to a deeper insight in the meaning and use of models and consequently made possible a more flexible approach in problem solving.

<u>Implications for Education and Future Research</u>

This study has provided useful insights into the effects of the experimental program on pupils' learning outcomes. The empirical findings go in the direction of the theory presented and as such are a confirmation of the hypothesis. Although the results are significant, the program did not have a strong influence on the pupils' mathematical achievement, as was shown in the regression analysis (3 percent of the variance was explained by the program, over and above the variance already explained by the pretest. The effect size was .40)

In this research, our resources were not sufficient to allow for more classes to participate in the experiment. Generalizing these results will be difficult, because of the limited number of classes (N=10). It would be useful to repeat this study with a larger number of classes.

This article did not address the interesting question from a longitudinal perspective, of how pupils' models developed in the experimental condition during the program. In future articles the way in which pupils make the step forward from a 'model-of' to a 'model-for' approach (Gravemeijer, 1997) will be examined in a more qualitative way.

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4. The learner as designer: Effects on transfer of an experimental curriculum in modeling[6]

[6] Van Dijk, I. M. A. W., van Oers, B., Terwel, J., & Van den Eeden, P. (submitted). The learner as designer: effects on transfer of an experimental curriculum in modeling.

Abstract

Is it better to provide students with teacher-made models or is it more effective to scaffold students in the process of designing their own models? In most strategy research the focus is on ready-made models provided by the teacher or a textbook. However, in this research project the effects are described of an experimental program in primary math education, concerning the construction and use of models by students in guided co-construction. Learning how to construct models is said to promote transfer. Therefore we expected students in the experimental condition to show a larger amount of transfer of the learned material than students in the control condition, especially with regard to more complex problems. In a field experiment, in which 238 grade-5 students were involved, this hypothesis was tested. In a series of experimental lessons, students were taught to design models as a tool in the learning of percentages and graphs. The scores on the transfer test of students in the experimental condition were compared to the scores of their counterparts in the control group, in which the students were exposed to the teachers' strategy of 'directly providing ready-made models'. It was concluded that students in the experimental condition significantly outperform students in the control condition in terms of transfer. In the analysis special attention was given to differential effects. However, no differential (interaction) effects could be found. Thus, both high and low achieving students profited from the intervention.

Introduction

In most strategy research the focus is on ready-made models provided by the teacher or a textbook. This article, however, reports on a research project aiming at describing and analyzing the effects of an experimental curriculum in primary mathematics education that emphasized the construction and use of models by students. Point of departure for this article is one of the major questions in learning theory and strategy research: providing or generating models and strategies? (Rosenshine, Meister & Chapman, 1996).

In a field experiment, students were taught to design models as a tool in the learning of percentages and graphs. In order to exceed the above-mentioned fundamental dilemma of Rosenshine et al. (1996) an experimental program was constructed and teachers were trained to assist students in designing models in a process of guided co-construction. In this experiment we compared the learning outcomes of students in the experimental program with learning outcomes of a control group in which ready-made models were provided. The experimental program is based on a vast body of empirical and theoretical research that demonstrates the potential relevance of guided co-construction as a teaching-learning strategy (Freudenthal, 1991; Hoek, Van den Eeden & Terwel, 1999; Van Dijk, van Oers, Terwel, & van den Eeden, in press; Van Parreren, 1993).

The research question focuses on how students cope with mathematical problems under two conditions and how they choose, use, and construct models. The major issue in this article, however, lies in the problem of transfer. Will transfer of knowledge and skills proceed to a different extent when students learn to collaboratively design models, in comparison to students who learn to work with ready-made models? The guiding research question in this article is as follows:

> What are the effects (in terms of transfer of learning outcomes) of an experimental primary mathematics curriculum, in which students participate as model designers in a process of guided co-construction?

The hypothesis is that the strategy of learning to design models collaboratively will have positive effects on the learning outcomes as measured by a transfer test in mathematics, due to the learning of strategies how to design models and the personal involvement in the reflective construction of the solution to the problems at hand. The basic idea underpinning this hypothesis is that students will gain deeper insight into math problems as a result of this constructive involvement and discussions. In line with the Vygotskian perspective on learning we assume that the students will gain a more reflective insight due to the interpersonal (collaborative) activity. We expect this deepened, reflective insight will help students to see through structures of new and unfamiliar problems. Hence, we may expect that the strategies acquired can be used to solve new problems and therefore enhance transfer.

Before moving on, the concept of transfer needs some clarification. What definitions are used to describe transfer? And how can transfer be enhanced? These questions will be addressed in the next section.

Theoretical Background

Transfer Theories

Some researchers, such as Alexander and Murphy (1999), define transfer as the process of using knowledge or skills acquired in one context in a new context. According to Larkin (1989), however, this definition is not correct. He argues that application of old knowledge in new situations occurs all the time. Think for example of searching a phone number in the phonebook. This means using the ability to read: a skill probably learned a long time ago. Therefore Larkin decided to define transfer as 'the application of old knowledge in a situation which is so new that it also requires the acquisition of new knowledge'. Larkin states that transfer leads to transmission of (some part of) former knowledge from earlier experiences.

It is important to distinguish two dimensions, when speaking of transfer: distance, and degree of generalization (Perkins & Salomon, 1989; Bimmel, 1999). Distance concerns the difference between the task trained and the task to which transfer has to take place. The polar adjectives on the 'distance' dimension are called 'near' and 'far' transfer. Near transfer takes place when students work on tasks that are seen as quite similar to the tasks used in training, and in a similar domain. Far transfer takes place when students are asked to use their knowledge in different sorts of tasks or in other domains than the tasks or domains to which they are acquainted.

The degree of generalization, on the other hand, pertains to the scope, or the reach of previously acquired knowledge and strategies. To put it differently, the degree of generalization corresponds to the number of specific tasks on which the learning outcomes can be applied (Marini & Genereux, 1995).

It is widely acknowledged that transfer of learning is a fundamental goal of education (Marini & Genereux, 1995). Therefore, teachers would like to see that students apply their learned knowledge and strategies in new situations. However, transfer doesn't always occur. Research by Verschaffel and others, for instance, demonstrated that children often do not transfer their 'outside' knowledge to school problems (Verschaffel, Greer, & De Corte, 2000). Verschaffel et al. provided a test with several standard word problems (S-problems) and several problematic word problems (P-problems). They found that in the P-problems most of the children did not use their everyday knowledge to solve these problems, and therefore gave unrealistic answers. For example: '450 baseball fans will go to the stadium by bus. Each bus can hold 36 fans. How many buses are needed?' Children tended to give answers like '12,5 buses', thereby leaving out the fact that half buses do not exist (Yoshida, Verschaffel, & De Corte, 1997). This example suggests that people, when confronted with new situations, usually apply knowledge and strategies that they previously experienced as useful in other more familiar domains. They tend to neglect the new parts of the situation and often inadvertently apply knowledge and strategies that are inappropriate for this new situation, thus generating negative transfer.

Common sense leads us to believe that 'when someone has acquired knowledge in one particular setting, it should save time and perhaps increase the effectiveness for future learning in related settings' (Larkin, 1989). But in the light of research on transfer, some claim that cases of significant transfer are 'rarer than volcanic eruptions and large earthquakes' and as difficult to predict. Alexander and Murphy (1999) argue against this view, by pointing out that transfer takes multiple forms with differing probabilities of occurrences. In their view, transfer isn't that rare at all! Moreover, they say: 'Not all transfer is as powerful as an earthquake and most occurrences of transfer go unnoticed'.

Transfer remains an intensively discussed topic in educational psychology (see for example Anderson, Reder, & Simon, 1996; Bimmel, 1999; De Corte, 1999; Greeno, 1997; Larkin, 1989; Perkins & Salomon, 1996; Simons, 1999; Van Oers, 2000). Discussions never resulted in one generally accepted definition of transfer. Though, these earlier discussions heavily influence the way we think of transfer these days, and therefore we would like to discuss some of these views in short.

One of the important ways of thinking about transfer in the twentieth century is the metaphor of the mind as a muscle. This theory suggests that, like a muscle, the more the mind is invoked, the more it develops and therefore less time is needed for the learning of new knowledge and skills (Larkin, 1989). In this view, transfer can take place between two totally different domains. When learning occurs faster in a new domain, it is assumed that this can be attributed to previous practicing of the related actions.

A reaction on this 'mind as a muscle' metaphor is the theory of Thorndike about identical elements (see De Corte, 1999). In Thorndike's experiments transfer did not appear to work across the board, and learning in one domain did not automatically speed up learning in another domain. Thorndike stated that only in the case of several identical elements between two tasks, learning in the second domain would proceed faster than the learning in the first domain. This assumption of identical elements is still a popular idea in explanations of transfer.

Greeno, Smith and Moore (1993) interpreted transfer from their 'situated cognition theory'. In their view, learners acquire an activity in response to constraints and affordances of the learning situation. Transfer of an activity to a new situation involves a transformation process. Transfer can occur when the transformed situation implies similar constraints and affordances as the initial context that are as such perceived by the learner.

The importance of the 'affordances' of the situation for the occurrence of transfer is also acknowledged in the work of the Dutch psychologist Van Parreren. In pursue of Allport; Klüver; and Katona, Van Parreren (1966) criticized the theory of 'identical elements' on two main points. Firstly, as a follower of Kurt Lewin's activity theory Van Parreren speaks of the 'valence' of situations. He preferred to speak of 'equivalent situations' instead of 'identical elements', while stressing the role of the characteristics and intentions of the learner in recognizing situations as equivalent as regards the actions to be accomplished. Thus, whether or not a situation is perceived as equivalent does not simply depend on the identical elements present, but depends also on the acknowledgement by the learner of the action possibilities of the situation given. Secondly, transfer does not occur automatically as the theory of identical elements suggests. Transfer is always connected to efforts of the learner. Transfer, in Van Parreren's point of view, has to be actively constructed by the learner. This point is picked up and developed further by Van Oers (1998; 2000).

From an activity theoretical and a sociocultural point of view, transfer is inconceivable as a simple transmission of knowledge (meanings) or views from one situation to another. Basically, transfer is held to be a process of transformation, in which knowledge is transformed to fit a new situation. Theoretically, however, the relationships between the old and the new situation need further explanation. In order to recognize equivalence between situations it is necessary to specify the symbolic-material basis for this process. Activity theory assumes that the inscriptions and symbols in a situation suggest possible meanings and support the process of transfer and transformation of associated meanings.

Conditions to Accomplish Transfer

One important question is whether the learning of general strategies can enhance transfer. Research pointed out that general principles of reasoning, learned in combination with self-monitoring practices and potential applications in diverse contexts, can indeed promote transfer. In contrast, Garnham and Oakhill (1994) state that knowledge and strategies that are taught in a certain domain usually transfer only within the boundaries of that domain. In this respect, like Garnham and Oakhill, we can refer to a review by Bransford, Arbitman-Smith, Stein and Vye (1985), which indicated that some important 'thinking skill' programs, like Feuerstein's Instrumental Enrichment or Lipman's Philosophy for Children, could not prove that the development of general skills led to transfer in a broad area of domains. No strong indications were found that students improved in tasks that did not match the tasks they explicitly practiced.

Thus far we have identified two broad conditions that seem to enhance the occurrence of transfer. One category has to do with task characteristics (distance) while the other is related to the quality of the acquired learning outcomes (degree of generalization). Several studies have yet exemplified how the organization of the learning process might influence the quality of the learning outcomes and, consequently, the quality of the learning outcomes that may influence the occurrence of transfer. Near transfer, for example, can be achieved on the basis of routines and skills that have previously been formed by frequent repetition of an insight or skill in diverse and changing contexts (Perkins & Salomon, 1996; Simons, 1990). Far transfer, on the other hand, is most likely to occur when students have been encouraged to get through the learning material in a productive manner. As Witteman (1997) argues: "Transfer of factual knowledge can only lead to expertise if learners adopt an active stance towards the incoming knowledge. Only the active handling of information will eventually lead to expertise. And only the learners themselves are capable of transforming knowledge to a strategic tool with which they can master a complex domain".

Verschaffel et al. (2000) mention an alternation of contextualizing and decontextualizing of the learning materials as a favorable condition for transfer. Starting out from a meaningful context for problem solving, students can be stimulated, according to these authors, to look for general principles or concepts that subsequently can be applied to new concrete problems. Reflection is an important element in these learning processes. This latter idea is confirmed by Davydov (1988) and Simons (1996).

In their studies with young children Brown and Palincsar (1989) specified the conditions for transfer still further. They showed that transfer of dd knowledge to new problems can take place when (a) learners are shown how problems resemble each other; (b) when learners' attention is directed to the underlying goal structure of comparable problems; (c) when the learners are familiar with the problem domains; (d) when examples are accompanied with rules, particularly when the latter are formulated by the learners themselves, and (e) when learning takes place in a social context, whereby justifications, principles and explanations are socially fostered, generated and contrasted. By taking into account these conditions, transfer can take place.

Yet another strategy that we found in literature for the promotion of transfer is letting students participate in an 'expert-culture' as much as possible (Collins, Brown & Newman, 1989). In an expert-culture 'practice situations' are created in which students have the chance to practice their skills on their own. Moreover, we may assume that these expert communities explicitly encourage and necessitate students to explore transformations of their already acquired knowledge and skills.

It is impossible to know beforehand whether learning results will be transferable. Only in new situations the proof can be visible: when students have to show and use their previously learned knowledge and skills. In sum, some of the conditions to enhance the occurrence of transfer are: productive processing of the learning material, alternation of contextualizing and recontextualizing; enhancing reflective activities; and participation in an 'expert-culture'. All these conditions are assumed to contribute to the quality of the learning processes and, consequently, to the quality of the learning outcomes. As such they contribute to the degree of generalization of the acquired actions. As we have said before, yet another condition is related to the perception of different situations as equivalent and as requiring actions or cognitive operations that are already available in some form. This latter issue is probably related to the material contextual cues (symbols, tools, inscriptions) that pop up in a situation. In our research we tried to elaborate this point by teaching students to see problem situations as equivalent with respect to the domains and the tools that could be used and (re)constructed. We assumed that the construction for oneself of flexible schematic models for problem solving contributes to both the recall of relevant knowledge (see Reese, 1977), and to generalization of the acquired actions. These conditions can help to bridge the distances between seemingly different situations. Therefore, we now turn to modeling as a thinking strategy that produces the means that may underlie transfer in mathematical problem solving.

Models to Enhance Transfer in Mathematics Learning

In our research, models are taken as tools for understanding that are supposed to enhance transfer in (primary) mathematics education. A survey of the educational literature provided us with several arguments underpinning this assumption, as well as critical commentaries warning against the overestimation of the power of models for learning.

In Dutch realistic mathematics education, models are promoted as a means to bridge the gap between concrete situations and abstract math. This can also be seen as a way of transfer, like Gravemeijer (1997) pointed out. He discussed different levels, following which models can develop. In the genesis of modeling he distinguished different stages, ranging from producing 'models-of' to the construction of 'models-for'. A 'model of' is a pictorial description of an actual situation with a high level of resemblance to the concrete situation itself. As a tool this type of model belongs to the category that Pierce would call 'icon' (Pierce, 1955). The 'models-for', on the other hand, are of a symbolic nature embodying mathematical content structures that make sense in their own right and that can be used for understanding the objects represented and even for deriving new knowledge about the objects represented.

Gravemeijer calls this evolution of models 'progressive mathematizing'. Eventually this process leads to algorithms, concepts and notations that are anchored in the students' personal learning histories. These outcomes are assumed to retain their intrinsic relationships with the real, informal, self-experienced knowledge from which they started. Actually, students transform their informal knowledge of a situation (and their models-of) into a new form that is more readily applicable in a new situation, and as such we can maintain that a process of transfer is involved.

Although we have good reasons to assume that models are useful for learning mathematics (Gravemeijer, 1997; Mayer, 1989), it is, however, doubtful that the imposition of ready-made models on students' thinking, will help them master the mathematizing activity (van Dijk, van Oers, Terwel, & van den Eeden, in press). From a sociocultural point of view, Wertsch (1998) contributes to the understanding of this dilemma by using concepts as mastery and appropriation. Mastery, in our view, can result from 'providing' practices and subsequent applying these ready-made models: it demonstrates "'knowing how' to use a cultural tool, without really making it your own". In contrast to the term mastery, we can clarify the concept of appropriation as a process of 'bringing something into oneself, to make something one's own' (Wertsch, 1998). The parallism of these notions with our concepts 'providing' and 'designing' is not hard to imagine. In both cases there is the distinction between transmission/reception on the one hand versus co-construction on the other.

The pros and cons mentioned above could be conceived as arguments for the idea that transfer depends to a great extent on the quality of the tools (Van Oers, 2000). Following Freudenthal (1991), we assume that the appropriation of the mathematical structuring ability might be more helpful than the mastery of mathematical structures. It was already pointed out that the mastery of particular strategies at itself isn't enough to enhance transfer. But what if you involve children in a social activity, like learning to design models? And let them gradually (re)invent what is earlier invented by others? "If the learner is guided to reinvent all this, then valuable knowledge and abilities will more easily be learned, retained and transferred than if imposed" (Freudenthal, 1991, p 49).

Many authors already pondered over this issue. DiSessa (1991) also wondered whether children could invent, in some reasonable sense, algebra, or decimal notation. His answer was 'yes': a child can become 'a designer of representational forms'. In his studies, he found that children were very well capable of inventing graphing. Although it seems impossible for children to 'reproduce in short order what took civilization thousands of years to build', diSessa claims that asking students to design models has several potential advantages. First, it closes the gap between prior knowledge and the material they are involved with. Second, it provides opportunities for creative engagement and ownership of conceptually difficult material (appropriation, Wertsch, 1998). And third, it lets students exercise meta-representational knowledge, which is expected to be of value for understanding any new representation. However, in spite of the advantages, diSessa noted "how rare it is to find instruction that trusts children to create their own representations" (1991, p. 156).

Taking the researches by Wertsch, van Oers, Freudenthal, and diSessa into account, we wondered if it might be more effective indeed if we taught students to design models themselves, in co-construction with peers and the teacher (Gravemeijer & Terwel, 2000). The idea behind this point of view was that students who learn to design models in co-construction would choose to make models that are more in harmony with their competence level and their day-to-day routine. Therefore it should help them to better understand the subject matter the model is about. Having learned how to construct adequate models presumably places the students in a better position for solving new and even complex or ill-defined problems for which no ready-made models are provided or available (Van Dijk et al., in press).

Many authors already have pointed out that when learning material makes human and personal sense to the learner, it is easier to use that knowledge in other (and different) situations (Donaldson, 1978; van Oers, 2000; van Parreren, 1966). Van Oers furthermore argued that it seems undeniable that some elements of culture (like models) can be handed over to other people. It is mainly the inscriptions (symbolic means) that can be transacted, but the meaning of those elements must be reconstructed by every individual in every new situation (van Oers, 2000). We therefore assume that models provided by the teacher make less ‘personal sense’ and therefore will be less easily used for new knowledge and tasks than models that are developed by students themselves.

Qualitative analyses of students’ model-based thinking (both reflection on models and evaluations) demonstrate that students indeed have informal ideas about models and their requirements. DiSessa (in press), for example, investigated students’ capabilities to deal with models, including the ability to design new models. High school students in this study appear to have a competence to judge representations. The students’ critical capabilities in diSessa’s study were characterized mainly in terms of the criteria they used for judgment.

Methods

Program Characteristics

In this study, the elements known in literature to enhance transfer were applied in an intervention program aimed at learning how to solve mathematical problems from real life situations. Especially students were assisted in their attempts to design models that represent a problem situation in such a way that it can be processed mathematically. Students were stimulated to:

1. Work in a productive and active manner in a setting that combined challenging, open problems with more reiterated tasks;
2. Contextualize and recontextualize the concepts and strategies in different contexts;
3. Share in class-wide and small group discussions how they dealt with the tasks;
4. Learn from each other by asking and giving explanations and to reflect on their ways of problem solving;
5. Learn by teacher demonstrations of several ways of designing models;
6. Work from informal representations.

The mathematical content was the same in the experimental and control conditions, but the main difference concerned the activity of modeling. In the control group a restricted range of models was *provided* by the teacher and the textbook, while in the experimental condition students were stimulated *to collaboratively design* their own models.

Although both conditions partially emphasized the same educational principles, some significant differences were made: Working in a productive and active manner meant in the providing condition that children individually worked with and drew ready-made models, whereas children in the designing condition collaboratively learned to design and develop their own models. Sharing the ways of problem solving in the providing condition meant that students discussed the way they worked with models provided to them, whereas students in the designing condition discussed their own models and improved them. The design of the research and the outcomes in terms of transfer of this intervention program will be discussed in the Research Design and the Results section.

Research Design

In this field experiment, with an experimental group and a control group, 8 schools, 10 classes, 10 teachers and 238 grade-5 students (age 10-11 years) were involved. A pretest-posttest control group design was used. 117 students were assigned to the 'providing' condition: teacher-made models were provided to the students while they were learning the percentage-concept. This 'model providing' approach is the way in which regular education takes place in most primary schools in the Netherlands. The experimental ('co-constructing') group, consisting of 121 students, was exposed to the same mathematical content, but here the emphasis was on 'guided co-construction' of models by students and teacher. This co-constructive learning approach can be characterized as a form of teaching in which the students participate as model designers in a mathematical context, and jointly construct mathematical models for the solution of complex problems. In earlier studies we found positive effects of this teaching method, like better learning results of students in mathematics (see van Dijk et al., in press).

The schools participating in the experiment were situated in, or near two cities in the center of the Netherlands. Experimental and control schools were either middle-class schools or schools with high proportions of ethnic minority group children. Both types of schools were equally represented in the two conditions.

The grade-5 classes were randomly assigned to the control condition or the experimental condition. There were no 'drop-outs' (schools, teachers, or classes) during the study.

Procedures

The experiment started with a workshop, for each of the conditions separately, in which the program and the teacher manual were explained and materials were discussed. The teachers participating in the experiment joined one of the two workshops. In the autumn of 1999 all teachers started the program in the same week and ended the program three weeks later.

The intervention consisted of a one-hour lesson every day for (almost) three weeks. It was composed of 13 lessons: an orienting lesson about models and their functions, and 12 lessons on percentages and graphs. For each condition a particular version of the program was made. These versions differed in the way students learned to work with models: in the experimental condition the students co-constructively learned to design models, as a tool for solving percentages problems. In order to strengthen the conditions for the development of representational ability (as needed for modeling) special attention was paid to the construction and use of graphs. In the control condition the students learned to apply ready-made models and graphs that the teacher provided. They did not learn to design or choose models themselves. More information about the tasks in the intervention can be found in Van Dijk et al. (in press).

All teachers were visited at least two times in the course of the investigation, to give them opportunities to ask questions about the material. These visits gave the researcher the opportunity to control the course of the lessons, the validity of the conditions and the integrity of the intervention. In addition, the teachers filled out a small questionnaire after each lesson.

Instruments and Analyses

We used a National standardized pretest and a special designed transfer test in order to find effects of the program in terms of transfer of learning results. The pretest and transfer test were equal for both groups. Pretest and transfer test proved to be reliable with alpha's of respectively .90 and .76. It should be noted that the alpha of the standardized pretest is based on oral reports of the Dutch National Institute (CITO) that developed the tests and that no publications are available yet.

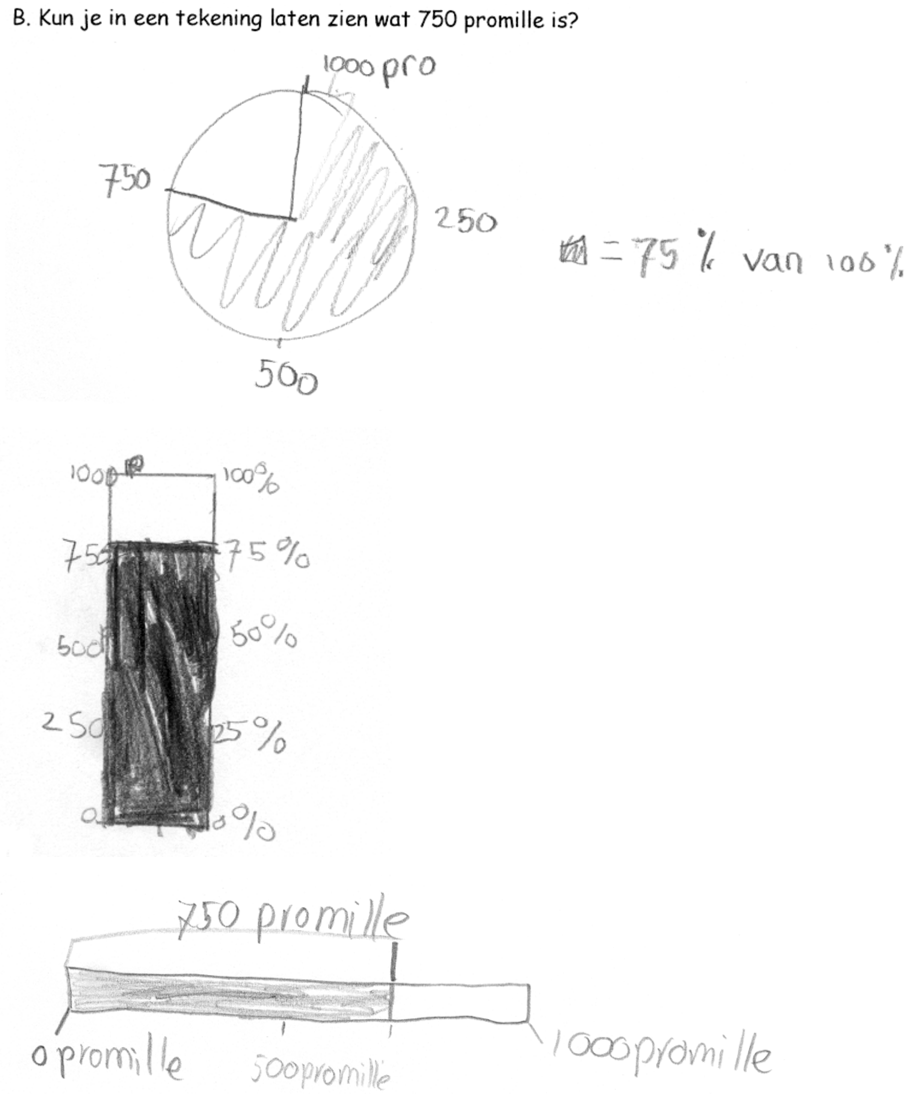

Figure 1: Examples of permillages tasks. Here, the students were just introduced to the permillages concept, by this text: 'Now you've learned what percentage means: 'something out of a hundred'. Something similar to percentage is 'permillages'. That means: 'Something out of a thousand'. The text above the assignment reads: 'Can you show what 750 per mille is, in a drawing?'.

The pretest (covariate) was meant to check students' mathematical knowledge and skills before the intervention started. Afterwards, the learning outcomes were measured by a transfer test with problems in situations and domains not closely related to what was taught in the lessons. The transfer test consisted of several items that were slightly (or totally) different from the kinds of mathematical problems the students worked on during the program. For example, students were asked to work with permillages, after a short textual introduction. The principles behind permillages resemble strongly the principles for working with percentages, but children need to reconsider their way of problem solving in order to solve the permillages tasks correctly. An example of students' work on the permillages task can be found in Figure 1. Some children made the connection between their existing knowledge of percentages and models, and used their models with some adaptations, as can be seen in Figure 1.

The transfer test consisted of 17 partly open and partly closed tasks. The test was meant to give information about what the students had learned of percentages and graphs. A series of tasks (7) in the transfer test was accompanied by a request to show the model that was used to solve the task. An observer scored these models and the outcomes were compared with the scores of a second rater for 30 students. Interobserver agreement was determined by Cohen's Kappa (.69). Resources like time and money were limited, so the possibility of a second observer for all cases was not feasible. Therefore the observer scored the models in a blind procedure. She did not know the name of the student, nor the class or intervention the student attended. A special identification number was attached to each individual test, which was at a later moment combined with the original respondent number.

The observer scored all models at four criteria, that were inspired by the works of DiSessa (in press) as was described in the 'Models to Enhance Transfer in Mathematics Learning' section. The four criteria were meant to represent criteria that appear to be important for a qualitative good model: structure, clarity, accuracy and completeness. In our view, a good model is a model that is accurate, neatly structured, conveniently arranged, and complete. Each individual model a student made was scored according to these four criteria. The score on each of the criteria varied from a minimum of 0 to a maximum of 4 points. After scoring each individual model, the scores of the models were added to the scores on the other items of the transfer test. The alpha of the transfer test was .76 (17 items). The results were processed and analyzed.

Table 1: *Characteristics of the Distributions of the Standardized Pretest and Transfer Test for All Students (N-Students = 238, N-classes = 10).*

	Mean	S.D.	Min	Max
Control program N-students = 117				
Pretest	136.07	22.02	49	189
Transfer Test	30.96	10.18	7.5	59.5
Experimental program N-students = 121				
Pretest	137.87	17.8	95	181
Transfer Test	37.38	9.94	10	60.25

Results

Outcomes

In order to determine the intervention effects, one-way ANOVA, regression analysis, effect sizes, and multilevel analysis were used. In Table 1 the characteristics of the distribution of the pretest and posttest are presented. In the process from pretest to posttest both groups show learning gains, but the experimental group gained more than the control group. From Table 1 it can be seen that the difference is about six points, in favor of the experimental group.

The effect size, as defined by Cohen (1988), was calculated. An effect size is the difference between the posttest means of the intervention group and that of the control group, divided by the standard deviation of the control group. The effect size of the transfer test is .63, which is a moderate effect. In educational environments, effect sizes of +.20 - .25 are seen as meaningful (Slavin, 1996, p. 31).

Analysis at Individual Level.

In a one-way analysis of variance no significant differences on the pretest scores were found (pretest: $F(1,237) = .484$, $p = .487$). However, a one-way analysis of variance resulted in significant differences between the conditions on the posttest (transfer test) in favor of the experimental group ($F(2, 236) = 138.135$, $p = .000$). Hence it can be concluded that in general there is a positive effect of the experimental program on learning results in terms of transfer. This result is in line with the hypothesis of the study.

We, then, considered the effects of the variables intervention and pretest on the transfer test outcomes. Here, no interaction effects were found. Therefore, a multiple linear regression analysis was conducted. The variables pretest and intervention were subsequently included in the equation. The outcomes are presented in Table 2.

From Table 2 it can be concluded that the pretest explains 46 percent of the variance in the posttest (transfer test). The intervention explains 8 percent over and above the variance already explained by the pretest. Thus, in this study we were able to explain 54 percent of the transfer test variance in total.

We decided to compare the regression equations on the transfer test of the two groups in this study. In Figure 2 we plotted the individual results of the children in the providing group and the experimental group on the pretest and posttest. For these tests we calculated the regression lines. The regression equations of respectively the experimental group and the control group are:

transfer test = - 17,025 + .395 pretest, and transfer test = -13,258 + .325 pretest. This means, in general children benefited from being in the experimental group.

Table 2: *Summary of Multiple Linear Regression Analysis for Variables Predicting the Scores on Transfer test (N=238)*

Variable	R	R^2	R^2change	Fchange	Sign R^2change	B	SE B	ß
Pretest	.681	.464	.464	205.1	.000	.353	.023	.669
Intervention	.734	.539	.075	38.6	.000	5.783	.930	.275

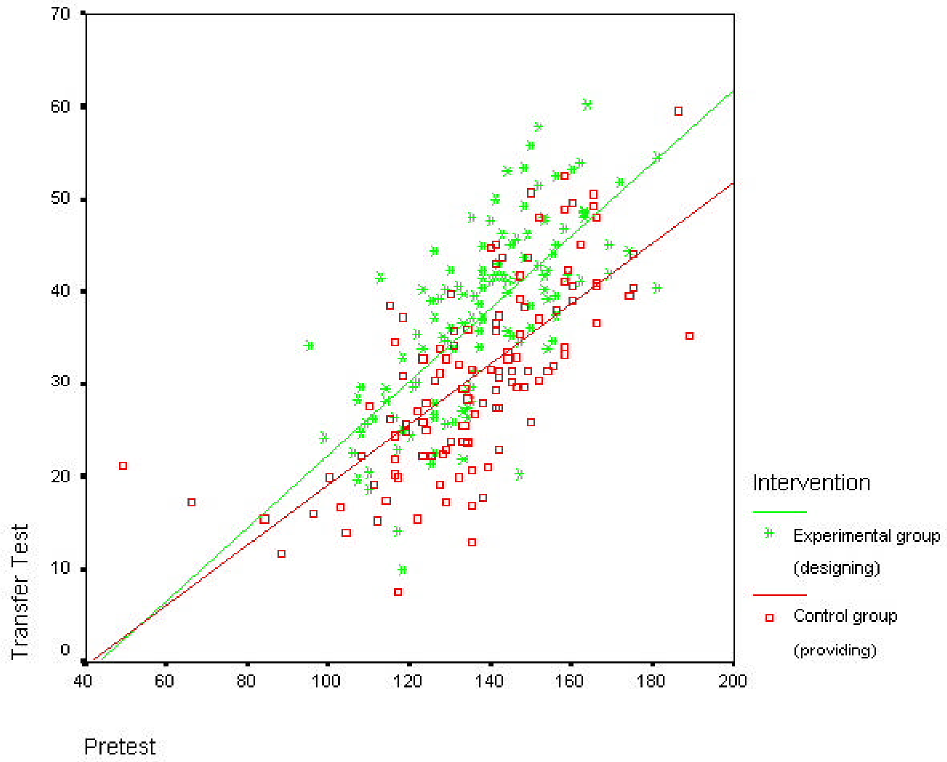

Figure 2: Comparison of Experimental and Control Group on the Transfer Test

After presenting the outcomes of the study in general, we will now look in particular at the more complex transfer items in the posttest. Earlier we pointed to some student work on permillages tasks (see Figure 1). This task, in which students are supposed to translate their knowledge of the concept of percentages into a model in order to handle a concept as permillages, is considered a more complex problem, since students have to transform part of their available knowledge for the correct solution of such problems.

The transfer test contained several of these more complex tasks. In order to get an impression of the way students dealt with these complex tasks, we composed Figure 3. We took the more complex tasks (7 in total), and contrasted the mean outcomes of the experimental group against the mean outcomes of the control group. Here, it can be seen that the students of the experimental group score higher at all problems, though with differing variance. Task 6A, for example, doesn't show a big difference: both groups have a reasonable score. However, especially the difference in mean scores on task 9b (the permillages task, as described earlier) is remarkable. While the students of the control group score a mean of 1,2 points (indicating that most of the students tried to solve the task with some model but that they were not very successful), the students of the experimental group score a mean of 2,7 points, which is twice as much (and indicates that most students received a score of 2 or 3 on a scale of 4).

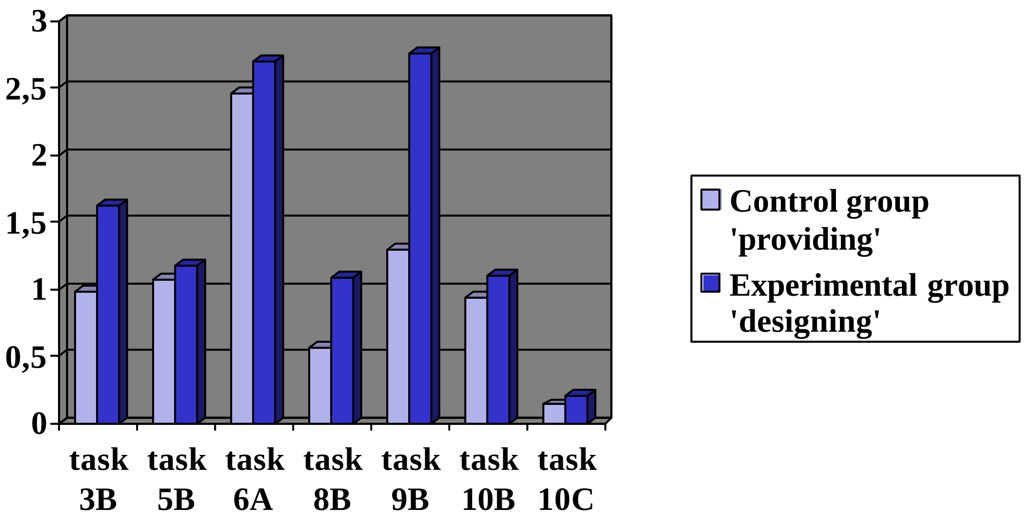

Figure 3: Means of complex transfer items for both conditions.

<u>Analysis on Individual and Class Level.</u>

This section deals with multilevel analysis to search for more detailed confirmation of our hypothesis. A multilevel analysis was executed to search in more detail for possible interaction effects, since it is suited for combinations of student- and class-variables (Terwel, Gillies, Van den Eeden & Hoek, 2001). Although the number of classes is limited for this kind of analysis, according to Snijders & Bosker (1999) this analysis is justified by N=10 at class level and n= 100 at student level. Our sample size is N=10 at class level and n=238 at student level.

Before presenting the outcomes of the multilevel analyses something has to be said about the meaning of the coefficients in Table 3. In Model 1, the variance in the dependent variable (transfer test) is divided in a component at student level (79.73 or 75 percent) and a part at class level (26.81 or 25 percent).

In model 2, three variables are introduced: pretest at student level and mean pretest and condition both at class level. The coefficients in the fixed part can be viewed as the conventional unstandardized regression coefficients. For example, in Table 3, the coefficient .33 means that a change of one unit on the pretest (pre-knowledge in mathematics) scale will result in a change of .33 units on the transfer test scale. The descriptives from Table 1 (Means) can be used to estimate the relative magnitude of the effects for an average student by multiplying the coefficient by the corresponding mean for the pretest.

Let us give an example of a class level variable from Table 3, model 2, by referring to the coefficient 5.63. Students in the experimental condition get a ‘bonus’ of 5.63 scale points on the transfer test.

The coefficients in the random part of Table 3 refer to the variances of disturbance terms that are left after introduction of the student- and class variables in the analysis. To put it differently, the random part concerns the residual (unexplained) variance after introducing all variables in the analysis. The variance left at student level is also called within-class residual variance. The variance left at class level is also called between-class residual variance. We present the outcomes of the multilevel analysis in Table 3. From Table 3, it can be concluded that after the introduction of the relevant variables, 48 % of the variance at student level and 75 % of the variance at class level was explained.

Table 3: *Results of the Multilevel Analysis, regarding the Transfer test in Mathematics as the Dependent Variable.*

	Model 1	Model 2
Fixed part		
Student level		
Pretest		.33 (0.2)
Class level		
Mean pretest		- -
Condition		5.63 (1.84)
Random part		
Student level		
Variance	79.73 (7.49)	41.76 (3.92)
Class level		
Variance	26.81 (13.52)	6.61 (3.76)
-2 * log (lh)	1732.15	1572.53

<u>*Note.*</u> *p = .05, Standard error between parentheses, N-Students = 238, N-classes = 10.*
‘-‘ = Non-significant effects.

In the multilevel analysis we, then, checked for slope differences between the classes. However, no significant differences between the slopes were found: the effects on the slopes are the same in all classes. Therefore we decided to look further for differences in intercepts between the classes to be explained by differences in class mean on the pretest (mean pretest) as well as condition (Program) The results of this analysis also show no effects for mean pretest.

Thus, on the basis of the multilevel analyses as presented in Table 3, we conclude that the experimental intervention had a significant, positive effect on the learning outcomes. No interaction effects for low and high achieving students could be found. Experimental students clearly show more progress in the learning of mathematics as compared to students in the control condition. The results of Table 3 are presented in a graph in Figure 4.

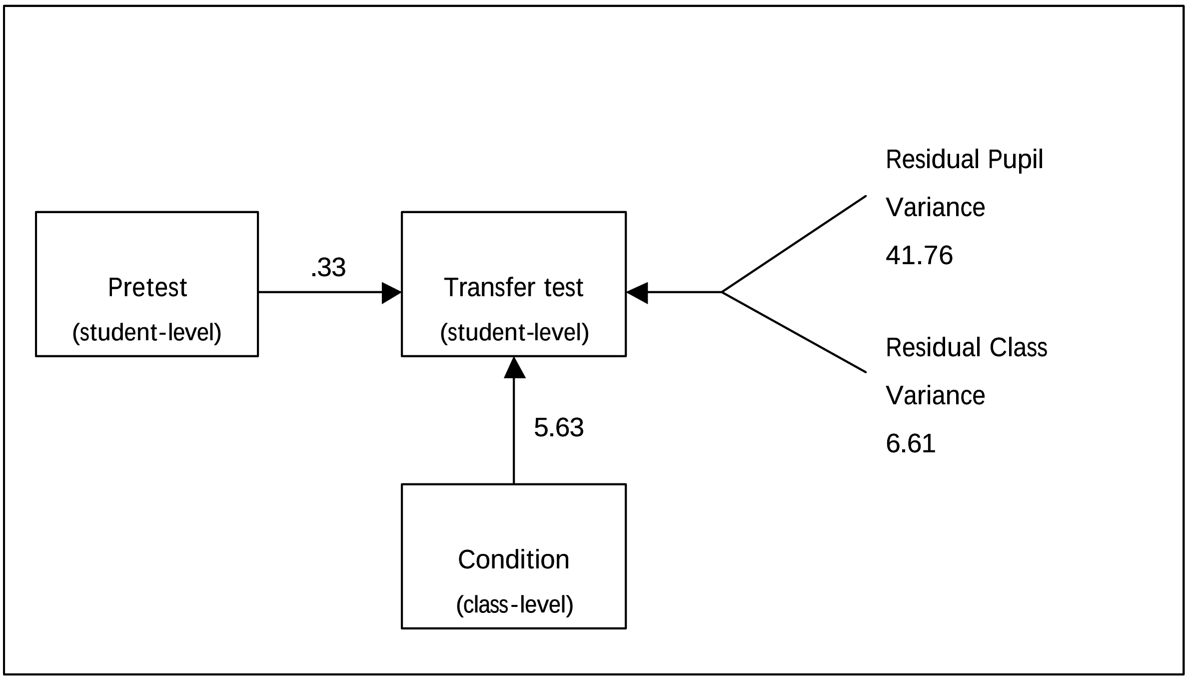

Figure 4: Diagram of the multilevel analysis

<u>Summary of Results</u>

We hypothesized that the students' learning processes in the experimental group would progress in such a way that a greater increase in results on the transfer test would be demonstrated, as compared to the growth of control group students. The results confirm our hypothesis: students in the experimental group significantly outperformed students in the control group on the transfer test. Taking the ANOVA, the regression analysis and the multilevel analysis into account, we can see that the results are significant and therefore we may conclude that the experimental program has a significant and practically relevant effect on the learning results (effect size .63). In the next section we will draw some conclusions. Implications for curriculum theory and practice will be discussed.

Conclusions and Discussion

Limitations and Conclusions

Before going to the conclusions something needs to be said about the limitations of this study. In this article we could not address the question on the learning processes. Extensive, qualitative descriptions of the learning processes will be presented in another article.

One could argue that this way of measuring gains of an experimental program by a transfer test is a kind of 'teaching to the test'. The students in the experimental condition could be said to have more experience with the process of model making than the students in the control condition. However, this critique can be refuted. Both groups should be in a position to produce models that obey the four criteria (structure, clarity, accuracy and completeness). Children of the experimental group, as well as children of the control group, learned to work with models, and both groups drew models during the whole intervention. The difference is that the control group drew models that they did not invent themselves, and therefore it may have been less easy for them to think of these models for the use in tasks that are not so closely related to the tasks in class.

In this article we addressed the question of transfer from a theoretical and empirical stance. Our research question was:

> What are the effects (in terms of transfer of learning outcomes) of an experimental primary mathematics curriculum, in which students participate as model designers in a process of guided co-construction?

On theoretical grounds we hypothesized that the strategy of learning to design models would be more effective than providing students with ready-made models. Therefore we expected a higher score on the transfer test in mathematics. In order to investigate this, we created different learning situations in the experimental group and control group that presumably differently affected the degree of generalization of the learned actions. In this article we could not address the question concerning the learning processes per se. Nonetheless, we did find significant differences in occurrences of transfer between the two conditions. Given the initial hypothesis and the analyses presented, the following conclusions can be drawn.

First, the outcomes of this study clearly show the expected learning results: significant and practically relevant differences in the outcomes between the experimental and control group were found. After controlling for initial differences in pretest, the program explained 8 percent of the variance in the outcomes on the transfer test with an effect size of .63.

Second, we found a relatively large effect of domain specific pre-knowledge on transfer. Pre-knowledge in mathematics as measured by a standardized pretest in mathematics, explained 46 percent of the variance in the scores on the transfer test. These outcomes are in line with Ausubel's well-known statement about the influence of pre-knowledge on learning. It is also in line with the literature about transfer, which indicates the domain specific character of transfer.

Although this additional finding urges us to be modest in our expectations about the effects of instructional programs on learning and (especially) transfer, we may conclude that students who learn to design models themselves score better on a transfer test than students who learned to work with ready-made models provided by the teacher. Learning to design models helps students to get more insight in the mathematical subjects of percentages and graphs and presumably also renders them more keen in the transformation of previously learned knowledge for new applications. It can be the case that students who learn to design models in co-construction choose to make models that are more in harmony with their competence level and their day-to-day routine. It should help them to better understand the subject matter the model is about. Having learned how to construct adequate models presumably places the students in a better position for solving new problems for which no ready-made models are provided or available.

As described earlier, we see our designing approach as an example of 'appropriation' instead of mere 'mastery' of knowledge (Wertsch, 1998). Strategic learning can be seen as a way of learning by appropriating tools. A model, used as a tool to bridge the gap from concrete to abstract mathematics, can be applied more successfully when students have an understanding of the principles by which it works, and have been able to develop a sense of ownership of the acquired knowledge through which they feel free to transform this knowledge according to the changing exigencies of the situation. We believe that children who learn to design are in a much better position to understand graphs, models or similar intellectual tools.

In short: the process of designing models can be a way of constructing tools for the solution of percentage and graphs problems and in addition enhance transfer to new situations. Therefore, in curricula and classroom practices more attention to the process of model designing could be useful.

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5. Development of modelling processes in primary mathematics: From informal representations to formal models[7]

[7] Van Dijk, I. M. A. W., van Oers, B., & Terwel, J. (submitted). The development of modeling processes in primary mathematics: From informal representations to formal models.

Abstract

Working with self-designed models appears to be an effective strategy to tackle complex problems in mathematics education. In a series of lessons, students were taught to design models themselves, in co-operation with peers and the teacher. Their informal models were taken as a starting point to gradually develop more formal models. Student work and videotaped lessons form the basis on which the process of emergent models is studied. An in-depth analysis is made of the developments of two students working together. It is shown that by learning to design, students make the transition from informal models to more formalized models.

Introduction

The use and importance of models in educational practices has frequently been an object of debate over the past decades. Learning how to create and revise models is seen as a central aim in education. Creating models provides a way to visually organize your understanding of complex phenomena. A model can be seen as a representation of reality, by which one can make statements about that reality. Particularly in the exact sciences the use of models is widespread.

But how do models emerge in students? Several authors tried to answer this question, by studies in areas as language-education, science and mathematics (Bednarz, Dufour-Janvier, Poirier, & Bacon, 1993; Carney & Kalathil, 2000; diSessa, Hammer, Sherin, & Kolpakowski, 1991; Forbus, Carney, Harris, & Sherin, 2001; Lehrer & Schauble, 1998, 2001; Mayer, 1989). Broadly seen, in these studies two approaches of model use can be identified: (1) studies in which ready-made models are *provided*, and (2) studies in which students are taught how to *develop* models.

Theories and studies endorsing this first approach are amply available (Hattie, Biggs, & Purdie, 1996; Mayer, 1989; Perkins & Unger, 1999). Following this approach, intervention programs are designed in which students learn to apply ready-made models. These models are seen as scientifically correct, formal tools, designed by experts in order to fit learning purposes. They are meant to shape students' transitions from concrete experiences to formal reasoning, as a ready-made scaffold to support abstract thinking. Students learn to apply the model in diverse situations, however, they sometimes cannot adapt the model itself or the way it was accomplished. By applying the model, and practicing the use of it, students are assumed to see the structures that lie behind the task, and in time they are eventually assumed to proceed without the ready-made model. Mayer's (1989) research on 'conceptual models' is an example of a 'top down' approach in which ready-made models are presented to students to enable understanding of the construction and functioning of technological systems like radar, computers, engines and brakes. In this view, learner-constructed models are not necessarily good (Perkins and Unger, 1999). A possible advantage of providing ready-made models is that they are developed by experts (e.g. teacher or curriculum designer), who are able to design models with 'future possibilities'. Such 'conceptual models' are designed with an eye on further introduction into scientific thinking and can be used in various situations and topics within the domain. However, although this kind of representations may support understanding, they do not warrant real understanding.

Unfortunately, it has been shown that not all children can see equally well through ready-made models (see Lampert, 1989; van Dijk, van Oers, & Terwel, in press[a]). In that case, the mathematical concepts embodied in the representations are only there for experts who already have those concepts available (Gravemeijer, 1997[b]). For students there is nothing more than the representation itself, but it has no personal meaning for them. As a result, the model cannot serve as a means to overcome the gap from concrete to abstract, and cannot fulfill the bridging function.

The second approach, although strongly embedded in constructivist and sociocultural theories, isn't widespread yet (Cobb, Gravemeijer, Yackel, & Whitenack, 1997; Gravemeijer, 1997[a], 1997[b]). As diSessa noted: 'how rare it is to find instruction that trusts children to create their own representations' (1991, p. 156). Not surprisingly, studies in which students learn to design their own models are scarce. In studies of the second approach, the active involvement of students is required for the accomplishment of models. In co-construction with other students and their teacher, students are able to construct models that are believed to fit their existing knowledge, daily environment and actual level. Students who have been involved in the process of model construction from the beginning, and see examples of emerging models, are assumed to better understand the structures that lie beneath the emerging model. As a consequence, they presumably will be able to apply it in other situations. This approach could be a solution for the just mentioned problems students experience when they do not understand a model or have no personal meaning attached to it, and therefore are not able to apply it correctly. But this way of working implies a risk: to ensure that the designed models are helpful, and are appropriate to extend on in the future, the teacher should keep in mind a few powerful models that can be used to confront the models of the students. The teachers should carefully monitor the developments of models. Several researchers experimented with students developing their own models and representations, either with help of computer applications (Carney & Kalathil, 2000; Kafai, Carter Ching & Marshall, 1997), or in other ways (Bednarz et al., 1993; Lehrer & Schauble, 2001; Mishra & Girod, 2000). Most of these studies focused on 'science education'.

However, in mathematics education too model construction processes are studied (Bednarz et al., 1993; Meira, in press; van Dijk et al., in press[a]). For example, the Dutch mathematician Dolk (1998) proposed to study the introduction of concrete contexts in primary mathematics, and to examine how models emerge out of these contexts. He took the development of the bar model as an example, which is a frequently used tool for the teaching of percentages. This bar model was introduced in several contexts, and generalizing and formalizing of the model by students was encouraged.

Our department carried out several studies on the topic of modeling in mathematics education. Regarding the question 'Providing or designing?' we decided to conduct a series of studies in which the two approaches were compared.

First, a small-scale case study was carried out. In this study, a 'providing' and a 'designing' classroom were contrasted. In order to explore the experimental conditions, two students, one of each classroom, were selected. They were followed closely in their learning process (van Dijk et al., in press[a]), focusing on the construction processes that occurred in a series of lessons involving problem-oriented tasks. Video tapings uncovered several differences in approaches of the two students. Overall, the student in the designing condition appeared to be more confident with her self-designed models than the providing student with her provided models. This case study clearly showed the intended activities of teachers and students. In addition, some indications were found that the expected results of learning occurred. Therefore, this study made it plausible to assume that children in the upper grades of primary school are capable of designing models in co-construction. From this qualitative study we received a firm foundation for the next study and a convincing case that the intended processes can be realized in normal classroom settings.

We, then, applied the experience and knowledge obtained in the explorative case study, to conduct a larger study (Van Dijk, van Oers, Terwel, & van den Eeden, in press[b]). In a field experiment, with an experimental group and a control group, 10 classes, 10 teachers and 238 grade-5 students (age 10-11 years) were involved. 117 students were assigned to the 'providing' condition. This 'model providing' approach is the way in which regular education takes place in most primary schools in the Netherlands. The experimental ('co-constructing') group, consisting of 121 students, was exposed to the same mathematical content, but here the emphasis was on 'guided co-construction' of models by students and teacher. This study should make clear whether the tentative conclusions stated as a result of the case study also hold for other topics, and a larger group of students. We were particularly interested in the effects on learning results of acquiring models by co-constructive learning as compared to the mastery of models by an expository teaching approach. So this time, we focused on the quantifiable learning outcomes.

Taken all together, the statistical analyses of the studies clearly show the positive effects of the experimental program. The conclusion, then, is that students who learned to construct models in the experimental program scored significantly better on the posttest and the transfer test than children who learned to work with models provided by the teacher. Learning how to design models in the context of a strategy for solving mathematical problems in daily-life situations is a valuable approach and puts students in a better position as compared to students who were provided with ready-made models. Our theoretical point of departure and the results of our experiment make us believe that designing models in co-construction may lead to a deeper insight in the meaning and use of models and consequently made possible a more flexible approach in problem solving.

In order to improve this approach even more for use in the future, we have to know more about the way the learning processes of students develop. However, until now little attention was paid to the process of learning to construct models, and emerging models. Therefore this will be elaborated in further detail in the underlying article, by use of videotaped lessons and student materials. We would like to examine the following research question:

> What are process characteristics of the emergence of formal models in primary mathematics education, under the conditions of co-constructive elaboration of students' informal models?

Theoretical Framework

Symbolic modeling is said to be central to mathematics (Lehrer & Schauble, 1998; van den Heuvel, 1995; van Oers, 1996[a]). In realistic mathematics education (RME), attention is paid to how students can play an active role in developing models and how models can evolve during the teaching-learning process, and – as a result of this – can prompt and support level raising (Gravemeijer, 1997[b]; Terwel, 1990; van den Heuvel, 1995). Therefore one might expect that it would be emphasized from the earliest years in elementary education. Unfortunately, in practice modeling isn't seen as a self-evident strategy that should be aimed for in primary mathematics (some exceptions put aside, see Davydov, 1988). Several explanations can be given for this fact. One possible explanation, stated by Lehrer and Schauble (1998), is the (ongoing) existence of educational theories that emphasize the need in primary education of focusing on simple, component parts for young students. In this view, these component parts develop gradually into more complex forms of reasoning, meant for older students. Careful observation of elementary school children's activities demonstrates, however, that this starting point doesn't hold in practice. In their spontaneous, everyday lives, young children deal with complex objects, images and concepts as a basis for their activity and learning (see Egan, 1999). According to Lehrer and Schauble (1998), early reasoning about models is anchored in children's invention and use of a broad variety of representational devices.

Concerning the definition of a model, in this article we will link up with Gravemeijer's view. In his view, a model is preliminary a 'representational format'. A model can be a drawing, a diagram, a symbolic representation, a way of notation, a story, or something like that (Gravemeijer, 1994, 1997[b]). A model can be a material or materialized construction, consisting of identifiable elements and relations, that structures in a way the actions of its user. Moreover, it is assumed that the model-based actions are equivalent to actions to be performed at another object (van Oers, 1999). Here, we confine ourselves to models that are observable in either curriculum materials, student materials, interaction and/or communication processes between students, or students and the teacher. A model, as mostly used in mathematics education, is a human construction: a tool designed by teachers or method-developers, either meant to make relations visible between phenomena or properties of phenomena, or to make a manageable representation of a complex procedure. Models can be seen as representations of problem situations. They can have different manifestations, but necessarily reflect essential aspects of mathematical concepts and structures that are relevant for the concerning problem situation (Van den Heuvel, 1995, p3).

All representational devices together, are called 'inscriptions': they include writing, drawing or yet another form of symbolization. Inscriptions find their origin in a more fundamental ability: being able to see an object as a representation of another and to reflect on the correspondences between object and representation (van Oers, 1994, 2000). Young children are at a very early stage capable to use this ability, as can be seen in their pretend play (see van Oers, 1996[b]). This strategy enables them to lay the foundation of "model-based reasoning". In simple arithmetic tasks, young children are able to use blocks, as representation of the task to solve it (see Hughes, 1986). At first sight, these blocks and the task have no relation, however, the blocks represent part of the task. But these basal representational skills are just the beginning of learning how to model. Modeling consists of several other aspects that are less easy to learn:

1. A model and the object the model refers to are at the same time equal as well as different. Although this is easy to realize for a mathematician, students often look at a model as if it is just a copy of the object the model refers to.
2. Sometimes, students are in consternation when there appears to be a discrepancy between the expected value of a measurement and the observed value. It is hard to understand that possible other models exist, and that an alternative model may be of more use than the model they handled until then. Differences between students in their reaction to discrepancies may be due to differences in tolerance of ambiguity.

Lehrer and Schauble (2001) argue that modeling should be practiced systematically in order to gradually discover the variety of forms and the advantages of certain models above other models in a specific situation. They state that students should be introduced into modeling with the help of models that have a strong resemblance with reality. These models, together with experiences with the situation, will support the mapping of reality. After some experiences with such modeling, students learn that the resemblance is less fundamental than the functional representation, and will be prepared to work with a kind of models that do no longer stick to similarities between the model and the real world.

Gravemeijer describes the implementation of instructional activities that began with children's understanding of an informal situation. Contextual problems are used as a starting point; these problems allow for a variety of informal solution procedures. As children mathematized these situations, they construct ways to communicate or model their thinking, first using pictorial representations and later using more formal notational methods, like equations. The applied problems precede instruction on the algorithms (Gravemeijer, 1997[b]).

Ideally, at first a model that is context-specific for the handling of a certain situation is developed ('*model-of*'). Then, generalization can take place and the model can be used in different situations, thus changing character and becoming a model that itself can serve as a tool for more formal mathematical reasoning ('*model-for*'). This process of '*model-of*' to '*model-for*' is called progressive mathematization (Gravemeijer, 1997[a], 1997[b]).

This process by which teacher and students take up notational methods can be explained as 'mediation'. The tools for this mediation include spoken words, written texts, (or representational models) which are passed on, and later transformed by individuals as they engage in legitimate activities in various contexts for learning. The more knowledgeable other, like the classroom teacher, is supposed to facilitate this process by introducing activities (e.g. scaffolding), so that novices may participate in and take up this cultural heritage (Whitenack, Knipping, & Novinger, 2000). This means that the 'classroom social norms' have to be explicitly (re)negotiated. According to Gravemeijer (1997[b]), the students have to become aware of the change in what is expected from them. They are no longer expected to produce 'correct' answers quickly. They now are expected to explain and justify their solutions, try to understand solutions of others, and ask when necessary for explanations or justifications by others. This shift will need a change in the role of the teacher.

But what are the advantages of a modeling approach, seen from the teacher's point of view? First of all, inventing, revising and developing of models by children makes their work and the way they think more visible than the traditional forms of question-and-answer and independent work (see also Lehrer and Schauble, 2000). Models leave traces of the students' cognition, especially when students work together in co-operative groups, and when they give each other explanations. This can be seen as a way of natural externalization in which thinking processes are brought to the open and thus are accessible for discussion (Terwel, Gillies, van den Eeden, & Hoek, 2001). Second, the process of modeling itself is valuable. It forces students to articulate relationships between entities and dependencies between their beliefs (Forbus et al., 2001). This is important for understanding the phenomenon being modeled.

Seen from the student's point of view, other advantages of a modeling approach are important. For instance, representations may help reduce working memory load, thus allowing students to work through more complex problems than they could otherwise (Forbus, et al., 2001). Furthermore, models provide a way to externalize thought. Representations help students present their ideas to others for collaboration and discussion. Peer-peer questioning, discussion and justification of ideas has been shown to aid learning (Forbus et al., 2001; Terwel et al., 2001).

Thus, a model works twofold: (1) as a scaffold, by showing others and the student himself how his thinking process developed and (2) as an elaborating means, by discussing, communication and confrontation cognitive conflicts arise, which makes level raising possible. Modeling can be even more powerful when students can use concepts they developed earlier, to tackle more complex problems. Van den Heuvel takes the bar model for learning percentages as an example. Her focus is on the emergence and evolution of the bar model as it is realized in the learning units of MIC (Mathematics in Contexts) concerning percentages. The intended process of model building is found to be in line with the student's way of working and thinking. In that way it enables them to reinvent the model by their own, or at least, to participate actively in the process of model building. Although it seems here that Van den Heuvel talks about the same subject as we do, (re-)inventing, a difference in approach can be found: we would like to start from the informal models children already have in mind without giving them a visual model in advance, while Van den Heuvel starts with a visual model that fits the children's starting level. During the process of growing understanding of percentages, the bar gradually changes from a concrete context-connected representation, which is going to function as an estimation model, to a model that guides the students in choosing the calculations that have to be made. In our results section we will show some more examples of emerging models.

The Design Approach in Education

Several researchers have adopted a design approach. Design tasks are often open complex problems (ill-structured) and afford many viable solutions. In this way design tasks are different from most tasks in classrooms, with problems that are well defined with clear-cut solutions. Design is a form of problem solving in which thinking, tool manipulation, and materials are reflected in the construction of an artifact (Mishra & Girod, 2000; Penner et al, 1998). In turn, artifacts become objects that facilitate the sharing of knowledge (Penner, Lehrer & Schauble, 1998). In fact, the thinking processes that proceed the designing of a model is a form of problem solving. Design problems are often rich structured, therefore the problem solver needs to define the problem at first, and later connect the problem definition with the proposed solution (see also Gravemeijer, 1997[b]). However, one has to bear in mind that a description of the problem situation in the form of schematizing and identifying the central relations by means of a model, does not automatically answer the question. It does simplify the problem by describing relations and distinguishing matters of major or minor importance. At least the translation and interpretation are likely to be easier, because the representations are meaningful for the problem solver, who is the one who gave them their meaning (Gravemeijer, 1997[b]).

As our point of departure in this article, we take the strategies of students to invent schemes in a progressive way for the inscribation of data. Activity by children leads to a wide variety of representations (or inscriptions, Lehrer & Schauble, 2001). One or more of these inscriptions develop as a means to objectify and refine descriptions of the world. These refinements themselves are the means to acquire more accurate estimations/reports of the world, leading to more inscriptional finishing touch. The circle is round. With projects like Lehrer's, and our own studies, students learn both about design – through the process of developing complex artifacts – and a variety of academic disciplines, such as mathematics and science (see also Mishra & Girod, 2000).

The preceding text and our view on working with a designing approach doesn't mean that representations provided by the teacher are by definition 'not done'. However, it does mean that in this case the children need space to build the representations together, supported by the teacher. The teacher still is highly responsible for evoking interactions and reflections necessary for that construction process (Nelissen, 1998).

Levels in Learning Processes

Students do their work at different levels of understanding; they pass through informal context-connected solutions, to some level of schematization, and finally having insight in the general principles behind a problem and being able to see the overall picture of it. Activity of mathematizing on a lower level can be subject of inquiry on a higher level: organizing activities that have been carried out initially in an informal way, later, as a result of reflection, may become more formal (Van den Heuvel, 1995, p2).

Several authors have mentioned the phenomena of levels in the learning process (Dekker, 1991; Freudenthal, 1973; Terwel, 1984, 1990; van Hiele, 1985). In most cases the levels were not only seen as descriptive categories, but also as a didactical opportunity to make transitions from lower to higher levels in the learning process. Freudenthal supposed to use the levels in the learning process as a way to accommodate individual differences between students in the classroom. In his view the principle of levels should become the major principle of differentiation within the heterogeneous classroom. Learning together in co-operative groups should enable all students to move from one level to the next. We will illustrate this process of level raising in the Results section (see for other examples Terwel, 1986). Although these descriptions and definitions by van Hiele, Freudenthal and Dekker are all valuable, in this article we chose to work with Gravemeijer's levels of activity, because his approach fits our purposes most. Gravemeijer (1997[b], 1999) argues that the development of a model-of into a model-for can be seen as a development through a series of activity levels, or from concrete to abstract.

Firstly, he distinguishes the *situational level*. At this level, situational and domain specific knowledge and strategies are used within the context of the situation to solve concrete problems. In this case we are talking about real-life activities. No paper and pencil tasks are involved. The models made by the students to solve mathematical problems are informal and are replicates of a situation that is familiar to the student.

The *referential level* is the next level Gravemeijer distinguishes. This level can be found mainly inside school situations, because the models and strategies used by the students to solve the mathematical problem refer to the situation described in the given problem. These models and strategies refer to the concrete situation that is experienced as real by the students and derive their meaning from its relation with the situation modeled. 'Models are initially tied to activity in specific settings and involve situation-specific imagery. At the referential level, situations are present in written tasks and models are grounded in students understanding of paradigmatic, experientially real settings. Students create models of the situation, and the situation still pervades the solution process. In whole-class discussions, these models are integral to explanations in which students describe how they interpreted and solved tasks centering on the starting-point settings' (Gravemeijer, 1999, pp. 163-164).

At the third level, the *general level*, models and strategies become more and more formal and students try to solve the problems from a more mathematical point of view, or as Gravemeijer states "a mathematical focus on strategies dominates the reference to the context" (1997[a], p. 30). The students don't have to think anymore about the situation sketched in the problem to give meaning to the models and strategies they use to solve the problems. The focus is on the strategy itself rather than on the context or the situation of the problem. The choice of a strategy no longer depends on its relation with the problem situation, but rather on the mathematical characteristics of the problem. 'General activity begins to emerge as students reasoning loses its dependency on situation-specific imagery, and the role of models gradually changes as they take on a life of their own' (Gravemeijer, 1999, pp. 163-164).

The fourth and last level is the *formal level*. When a student reaches this level, he or she is able to solve a mathematical problem with mathematical reasoning. The model used by the student is a model for mathematical reasoning and no longer a (concrete) representation of the situation sketched in the problem.

"This transition (between levels of activity) can be seen as a process of reification (Sfard, 2000), wherein the students begin to reflect collectively on their referential activity. In the process, the model becomes an entity in its own right and serves more as a means of mathematical reasoning than as a way to symbolize mathematical activity grounded in particular settings" (Gravemeijer, 1999, pp. 163-164).

In sum, this is what we see as the ideal way to teach students to work with models: students should learn to model mathematical situations and design applicable models, in co-construction with peers and the teacher, starting from informal representations (models-of) which will be scaffolded into more formal, abstract models (models-for). The distinction between the levels described above can be illuminated on the basis of the models the students in our study designed. The distinction between a model-of and a model-for can be seen as the transition from the referential to the general level. At the referential level, the model refers to the situation as described in the problem. The model has some kind of meaning for the student because of the reference to the concrete situation, which is understandable for the student. When the student has more experience with working with these kinds of models, the attention can shift towards the mathematical relationship in the problem. In the next sections we will elaborate a study we conducted, in order to exemplify details of the emergent modeling.

Methods

Design, Participants and Curriculum

This case study was embedded in a larger study, which was conducted with 10 classes, 10 teachers and 238 grade-5 students (10-11 years old). 5 of these classes participated in the providing condition (e.g.: ready-made models, provided by the teacher); the other 5 classes participated in the designing condition (e.g.: re-inventing and designing models in co-construction with peers and teacher). In order to acquire in-depth observations, we decided to select one classroom of the experimental condition out of the five classes. Therefore the data used in this particular article are collected at a primary school, which participated with a grade-5 classroom and the teacher. This school is situated near the center of Amsterdam, and is characterized as having a middle-class population and about 20 % of students from ethnic minority groups within this classroom. We selected two students, who were by the teachers judged as average students in mathematics. These students are Lisa and Tess. Lisa and Tess are friends and get along well. They are about the same age, respectively 10;4 years and 10;7 years old, and are both having a Dutch white middle class background. The classroom to which Lisa and Tess belong, consists of 24 grade-5 students. The two girls are seated in the same table group and like to work together. Lisa is a slightly built girl, she is calm, pliable, easy-going and sometimes appears to be unsure of herself and her abilities. Tess, however, has a ready tongue and is a more dominating type. She is a slightly better student than Lisa, and has enough self-confidence to make herself heard. Tess seems to be the one that likes to take the lead, but Lisa is the more reflecting person, asking questions like: ‘are you sure’?

According to the teacher, both students are average mathematics students. However, as compared to the overall mean of all students (N=238), Lisa scored somewhat below average on the pretest while Tess scored somewhat above average. On the posttest Lisa's score was average, while Tess scored significantly above the overall mean. On the transfer test Lisa scored slightly above the overall mean, whereas Tess scored far above the overall mean.

We followed these students closely. Our analysis in this article is based on videotapes of these students, and is amplified with observations of class discussions and remarks by other students where possible. Because teachers in the experimental condition were explicitly trained to promote the process of designing models and level raising, we expected most developments in level raising to occur in the experimental condition. For practical reasons, we confined ourselves only to this condition in our present analysis.

The tasks and assignments to be done in the lessons were mostly open, complex problems. They were chosen from current math methods used in the Netherlands (Pluspunt, Wereld in Getallen Nieuw) and in the United States (Mathematics in Context). Furthermore, exercises were used which had been developed for MILE (Multimedia Interactive Learning Environment), a project of the Freudenthal Institute for teacher trainees. All the exercises used were adapted to fit the purposes of the intervention intended. In addition, a teachers' manual was developed containing the main ideas and principles of the approaches.

The intervention consisted of 13 lessons on percentages and graphs, and took almost 3 weeks. During the lessons, students worked with two books: a book in which the assignments were given, and a book in which they could work out their answers. These workbooks were collected at the end of the intervention, thus providing valuable information about the way the students worked with models.

<u>Procedures, Instruments and Analyses</u>

The teacher of the group joined a workshop with others teachers of the experimental condition, in which the starting points of the condition concerned were mapped out. During this meeting they had the opportunity to work with the new materials themselves. A couple of weeks later, the intervention started and all the intervention lessons were videotaped, thereby focusing particularly on two children.

In this section we will further elaborate on the procedures a teacher could follow in order to stimulate students to design models in co-construction. After introduction of a complex problem, the students were asked to work in pairs on the task. It was emphasized that there was no single right answer, and that several solutions could be useful. Students were asked explicitly to make representations of the problem within the context of the total problem solving cycle. During the work in pairs, the teacher walked around the classroom, looking for useful or original representations, and asked several students to draw their models on the black board. These models and ways of solving the problem were then discussed class-wide, in which (dis-)advantages of aspects as speed of representation, accurateness and completeness of each representation came to the front. Students were invited to comment on the models, or suggest corrections or additions. After these discussions, they could draw their conclusions, and move on to a next assignment in which the new insights could be tried or applied.

Another frequently used way to let students learn from each other's approaches, was asking them to walk through the classroom and take a look at the models of others. The students were invited to ask each other questions, and talk about possible ways of improving the representations. Then, in a short class-wide session, the teacher asked them what representations they liked in particular and what they liked about it. In both ways, reflection is well underway. Important in this approach is the role of the teacher: instead of transmitter of knowledge, he becomes a critical guide and participant in the process.

The videotaped lessons formed the basis of our analysis, together with the collected workbooks. After observing all lessons, we decided to focus on a couple of tasks, because these tasks would reveal most of the processes we were looking for. By means of the videotapes and the workbooks, we tried to connect Gravemeijer's levels of activity to the actual classroom practices. We searched for episodes that illustrated the levels of activity, and the developments of students, e.g. the shifts they made between these levels. As mentioned, we focused on two students in particular, and added protocols of class-wide discussions about models designed by other students.

Results

We followed two students (both girls, Lisa en Tess) through their efforts and developments during the lessons. The situational level can hardly be found in these lessons: though strongly embedded in contexts, the tasks are presented as paper and pencil tasks, and not as real-life activities. But in the series of tasks several shifts can be seen from referential level to general level. In this section we will exemplify Gravemeijers' levels of activity with concrete examples of work and discussion by Lisa and Tess and some of the other students in the same classroom.

<u>Referential Level, or Model-of</u>

The goal of the first lesson is to check what the children already know about percentages. The informal knowledge is being used as a starting point in the first task. A story is told about Evelyn and Peter, who brought a dartboard from home. Evelyn says: "When I throw a series of 20, I usually score 4 bull's-eyes". Peter thinks he's a better player. He says: "When I throw a series of 25, I usually score six bull's-eyes". The children standing around them butt in on the discussion. One girl thinks Evelyn is a better player, while another girl thinks that Peter is better. The question is: who do you think is a better player? Tess and Lisa are trying to capture the task about the darting kids in a model (see Figure 1).

Lisa: I'm not sure how to show it… Do you get it?

Tess: We ought to draw something that shows that Peter is the best. Each time Evelyn throws 5 darts, one of them is in the bull's-eye.

Lisa: Oh, so she throws 5 darts, and one is in the bull's-eye. So if she throws 20 darts, she throws 4 bull's-eyes.

Tess: Look at this. Peter throws… eh… how many times 6 is 25? Well, 24 is 4 times 6. So, Peter throws one dart in the bull's-eye, out of every 4 darts, and he has then one dart left. Peter is better!

Lisa: But how do we show this in a drawing? That's difficult.

Tess: Let's just draw a dartboard and put their names on it. Here, I draw 4 darts wide, and one in the bull's-eye for Evelyn, and I draw 3 darts wide and one in the bull's-eye for Peter.

Lisa: Yes, Peter is a better player!

In a class discussion, concrete models like Lisa's and Tess's come along. In our view, these models have several characteristics that refer to the referential level: In all models a strong connection to the situation can be found: dartboards and darts are amply available, but from most models it is difficult to decide who is a better player. Furthermore, these models are only applicable on this particular situation. In a task about discount in a shop, this model with dartboard and darts will not fit.

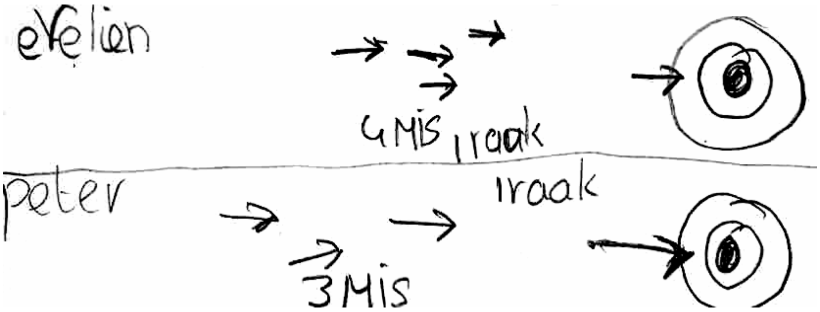

Figure 1: Model of the dartboard task, designed by Lisa. The text she wrote says: 'Evelien, 4 misses, 1 bull's-eye' and 'Peter, 3 misses, 1 bull's-eye'.

However, one of the students has another solution. Todd draws two lines, one for Evelyn and one for Peter. Evelyn's line is a bit shorter, because she only throws 20 darts, and Peter throws 25 darts. Todd divides Evelyn's line in 4 equal pieces and explains:

> "Every time Evelyn throws 5 darts, she throws one bull's-eye. Therefore above every piece of the line I note: '1x'. Peter's line should be divided in 6, because out of every 4 darts, he throws one bull's-eye. So above every piece I note '1x'. If we ask Evelyn to throw 5 darts more, she throws as much darts as Peter. So I make her line a bit longer, as long as Peter's, and she will hit the bull's-eye once more, I guess. Then I know who's better: Peter throws 25 times, and has six hits, and Evelyn throws 25 times and has five hits". (See Figure 2)

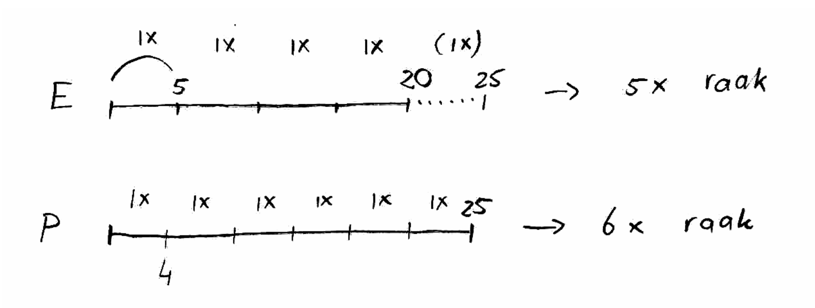

Figure 2: Model of the dartboard task, designed by Todd. The 'E' stands for Evelyn; the 'P' is for Peter. After the line of Evelyn he wrote: '5 bull's-eyes', and after the line of Peter he wrote: '6 bull's-eyes'.

This model has some characteristics that refer to a more general level. In this model no concrete references to the situation are made. Todd uses a simple, empty line in combination with (mathematical) symbols to represent the number of throws and the number of bull's-eyes. He could easily adapt this model for use in another task.

The teacher re-voices Todd's approach to emphasize the importance of making the situation comparable. Todd chose to make it equal to 25, but he could also have chosen for 100. The teacher explains to the children that some time ago, people invented a way to compare different things and that this way was called 'percentages'. He then explains that percentage means 'something out of a 100', and makes the connection to fractions, with which the students are already familiar.

The second task handles the relativity of percentages; you always take a percentage out of something. The students are stimulated to discover this by themselves, and verbalize it. They are supposed to give their opinion on short stories about percentages, on the basis of their existing knowledge. One of the short stories is about runners in a race, and two children who are talking about this race. One of them states that half of the runners have dropped out, and the other child says that she agrees: about 60 percent of the runners have stopped. This task provokes a conflict for Tess and Lisa. The children in the task explicitly say they agree with each other, but to Tess and Lisa they seem to say different things. Our two students decide to represent the runners by puppets, to see what the children in the task mean. The opinion of the one child is represented by 3 puppets lying down (indicating that they have dropped out of the race) and 3 others still standing upright. The opinion of the other child is then represented by 4 puppets lying down and 2 puppets standing upright. Amy draws her model at the black board: for the one child (Vera) she draws 10 circles and shades 6 of them, and for the other child (Jolanda) she draws 10 circles and shades 5 of them.

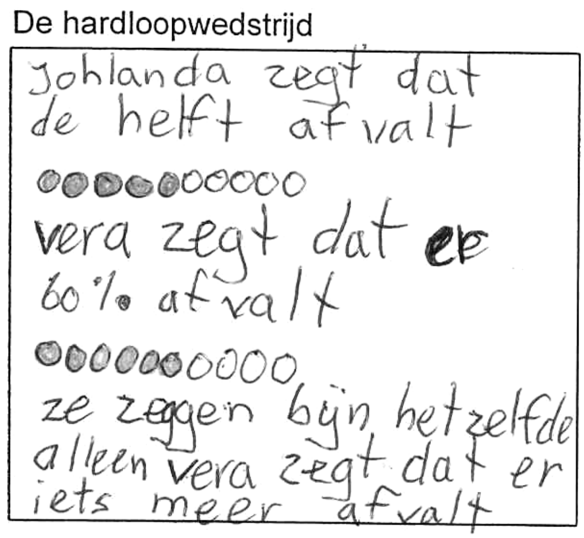

Figure 3: Model of the running race, designed by Amy. She wrote down: 'Jolanda says that half of the runners dropped out. Vera says that 60% of the runners dropped out. They say almost the same, but Vera says some more runners have dropped out'.

Amy: Well, now you can see that 60 % is only a little bit more than a half.

Teacher: Very good! The difference between 60 % and a half, 50 %, isn't a very big difference. You said it right: it's just a little bit more.

By means of Amy's model and her explanation, the teacher emphasizes why the two children in the task agree with each other, while they seem to say different things. Following this model (see Figure 3), the students now all agree that 'half of the runners' and '60 % of the runners' is almost the same.

The students then moved on with an assignment about two children, named Sarah and Anouk, who bought the same scarf. But when they saw each other a couple of days later, strange things have happened. Sarah's scarf was only 50% of it original length, because it crump in the washing machine. However, Anouks scarf was 200% of the original length, because of her dog had been pulling it. When asked to represent this situation, most of the students made a drawing that is close to the situation. Lisa and Tess did too. In fact, their model isn't even a model at referential level: it is just a very nice drawing of a dog pulling a scarf and of a washing machine. However, the most important parts of this task, which is the difference in length of both scarves, did not show in their drawings. One of the other students gets the opportunity to show his solution on the black board. He draws the following figure (see Figure 4).

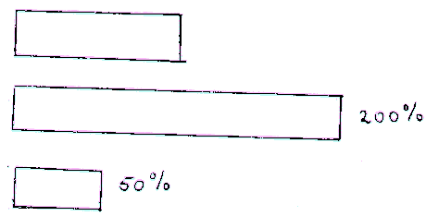

Figure 4: Model of the scarf task, designed by Noah.

This model seems to be at a general level: this student represents only the result of the situation. He did not draw the circumstances which made the scarves change, nor did he draw concrete scarves with all kind of details. Furthermore, he added some useful notes like 50 % and 200 %. He calls one person 'A', and the other person 'B' when he explains his model to the rest of the group. In fact, he shows to understand that names and circumstances are not that important when designing a model. With only little adaptations this model can be used in other tasks.

The next lesson, the students are asked to draw percentage stories. The first story says: '50% of the students are girls', in the second story it says '25% of the flowers are red'. Comparing the drawings of several students, it is remarkable that several students make drawings that resemble exactly the boys, girls, red flowers and cookies. However, in drawings by some other children of the group no direct references to the concrete situation are found. For example, they choose to shade part of the box (which is given as a sort of scribbling-paper) in order to make clear what 50 %, 25 % or 100% means (see Figure 5). It seems that these students are able to make representations at another level, without concrete references to the situation in their models. The models described above are added in Figure 5.

When we take a closer look at the figures Lisa and Tess designed until this point, it is notable that these figures look very much like the reality and the information that is given in the task. Their drawings show a dartboard and darts, or flowers, or a dog and a washing machine, all referring directly to the situation. The model has a meaning for the students in a way that it refers to the concrete situation. However, the model only depicts the situation and does not go beyond the information given. We would describe this kind of models as models on the 'referential level'.

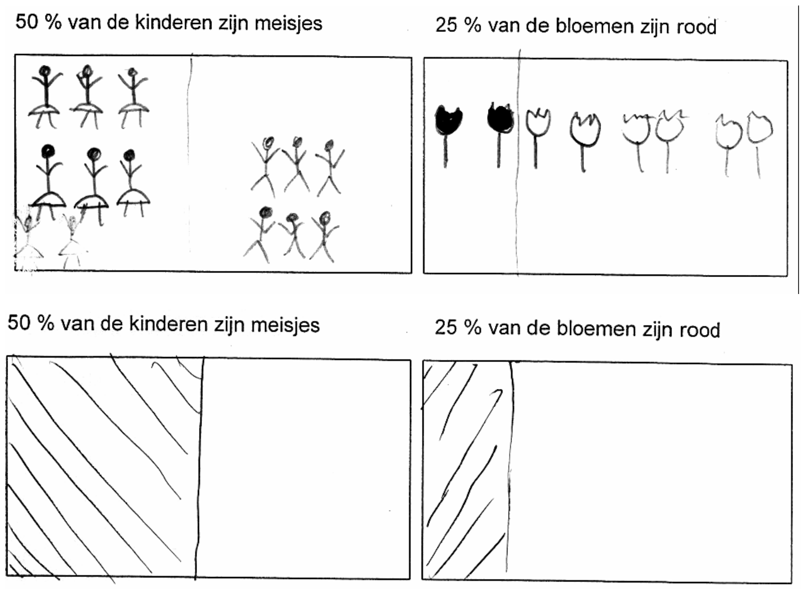

Figure 5: Model of the percentages stories. The text says: '50% of the children are girls' and '25% of the flowers are red'. The first one is made by Tess; the second one is made by Pete.

In interaction and by discussing with peers, models should grow and develop to a higher level. In most cases we saw until now, the model is context-specific for the particular task ('*model-of*'). Disadvantage of these models-of, is that they can only be used in that particular situation and that they can not easily be applied in other tasks with another context, where the same solution strategy could be useful. These models-of have a close resemblance to the situation, and they can serve as a means to connect with informal strategies in order to make students aware of these strategies. They are the stepping-stone for the gradual evolution of more formalized models.

However, even at this early stage some students show models that appear to be on a higher level. Todd's model, and the scarf model are models that can be used in various situations. We can say these models are on a more formal level: a general level, which means that no direct connection is visible to the specific characteristics of the context of the task. The drawing of the scarf could also be used for a any other task in which one has to represent that something became twice as big, or half of what it used to be. Thus, in the discussion with the whole class we see a glimpse of a process of de-contextualization, or level raising from referential level to general level between a group of students.

But, how does this development proceed, from a referential level to a general level? The first lessons Tess and Lisa appeared to be on a referential level. They made 'models-of'. Developments from a referential level to a general level, and discussions of Tess and Lisa will be discussed in the next paragraph.

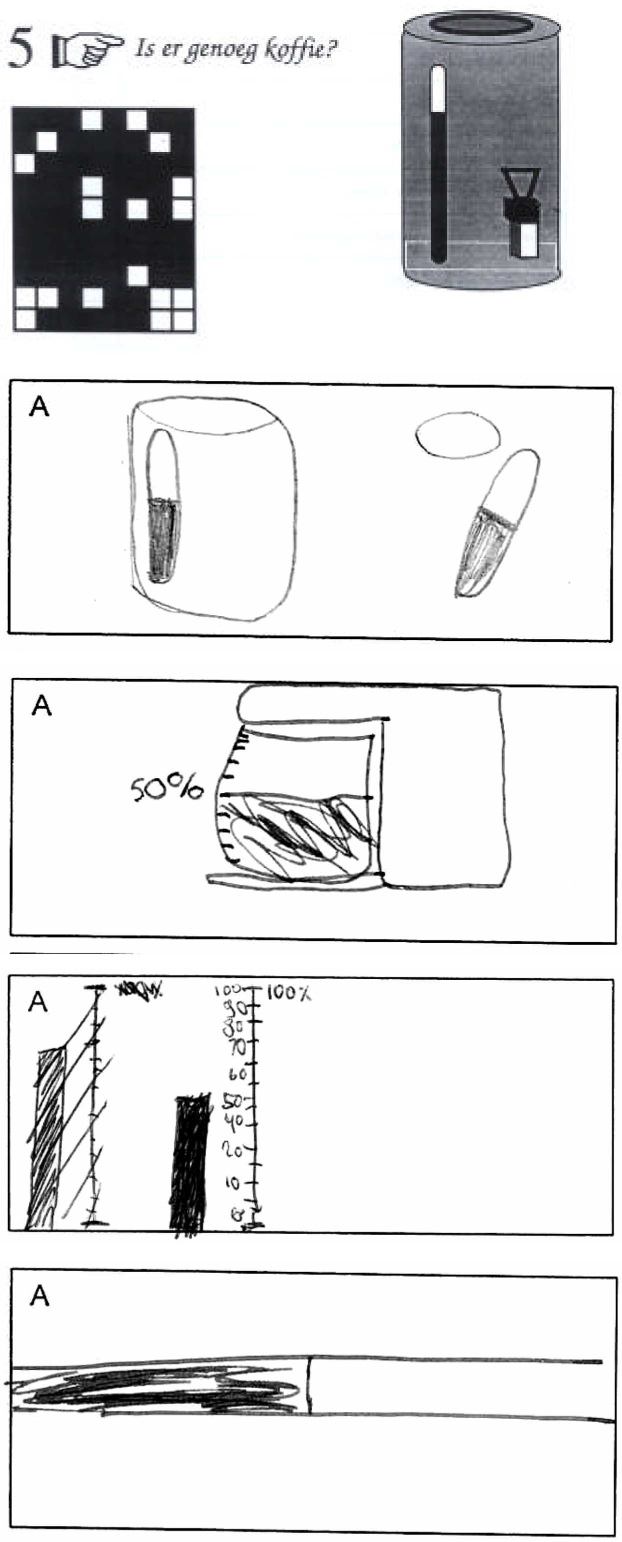

Figure 6: Model of the coffeepot task. The text above the task says: 'Will there be enough coffee?'. The models are designed by respectively Lisa, Eric, Yil and Walter.

A Development from Model-of to Model-for

The assignment about a coffee pot is an open-ended and ill structured task. An extended description of the assignment can be found in van Dijk et al. (in press). Here we will just focus on the last part of it. The question is whether there is enough coffee left to provide everyone with one cup of coffee. About three quarters of the pot is filled, which means that about 60 cups of coffee are available. The students figure out that this is not enough coffee for the 61 coffee-drinking people.

Lisa is invited to show her model at the black board. She tells the class that she made a coffeepot, with a gauge-glass in it. At the black board a rectangle arises, and a narrow, vertical bar is drawn in this rectangle. She shades the bar for about 25 %. The teacher uses Lisa's model to solve the task. With some help, Lisa is able to tell how much cups of coffee are left in the pot when it is for 25 % filled. In words, her model is obviously strongly context-bound, but in her drawing her model doesn't really look like a coffeepot.

Eric, however, draws a real look-a-like coffeepot at the black board. Yet two other students draw a model at a more abstract level. Figure 6 shows the models these students came up with.

Tess seems to prefer one of the models she worked with in the coffeepot assignment. She drew two models, the first was a rectangle with a bar in it, like the one Lisa showed at the black board, and the second one was a bar on it's own. In the next task she spontaneously applies this second model again. This time the task is about a boy who has to pay 25 % more rent, and who asks his mother for money. The teacher notices her model use and asks her to show the other students what she did (see Figure 7). Tess draws a bar, shades a quarter of it, and explains that this boy has to pay 25 % more, out of 200 guilders. Pointing to the shaded part, she says: 'he has to pay this 25 % amount twice', thereby indicating that the total amount he has to pay is becoming 25 % more than the original amount. The teacher then helps her to draw it right: first the total amount of 200 guilders, and then another 'extra' 25 %. Although the two tasks show no similarities at first sight, Tess finds it useful to draw a similar model in both cases. This can be seen as an indication that she moves from referential level to general level.

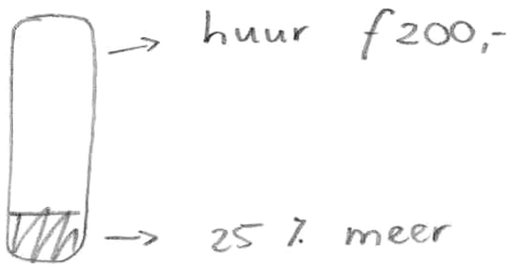

Figure 7: Model of the rent task, designed by Tess. The text says: 'rent 200 guilders' and '25% more'.

In the next lesson per group the students receive a power check battery. They are asked to take a look at it and to try how it works. At first, Tess finds the text on the battery very appealing. She tries to understand what it says and reads it to the others in all the languages that are available. Lisa immediately notices a '100%' sign on the battery, but the others are too busy discovering the battery for themselves and do not hear what she says. Together, they discover that if you push two small white buttons on the battery at the same time, a thin bar on the battery turns almost completely yellow.

The teacher asks the students what they think 'power check' means, as it is an unusual expression in the Dutch language. Summarizing the students' answers, the teacher states that with a power check battery you can check how much power, or energy, is left by pulling two buttons at the battery. A 100 % yellow bar means it is completely filled. Then, he tells them that energy corresponds with time: a battery can be used for a certain amount of time. How long a battery will work depends on the sort battery and where you use it in. This introduction is the context in which the students start working with several tasks about batteries.

The students have to make a drawing that shows how much energy is left in the power check battery they just studied together. Tess and Lisa exactly duplicate the battery (see Figure 8). Even the colors they choose resemble the real battery.

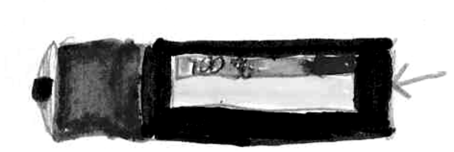

Figure 8: Model of the power check battery, designed by Tess and Lisa.

To try to encourage students to make connections between tasks at a higher level, the teacher introduces an extra, short task. He makes a connection between this battery task and a task of the last lesson (the coffee-pot), by using a dynamic 'slide bar' which has appeared earlier. A slide bar is a sort of rectangle with a window, through which a dynamic sliding bar is visible and can be manipulated. This sliding bar is half red and half yellow, and the transition from red to yellow can be moved from left to right and back in order to show a certain amount of percentages. The teacher asks the students questions like: "In the coffeepot task, what did the red part of the bar mean, and what did the yellow part mean?", "how could you relate the red part with the amount of cups of coffee?". Then he takes a shift to the battery task: "how much % is filled in this battery bar? And how many hours is that?", while he indicates a yellow part of 90 % and a red part of 10% at the slide bar. Tess is allowed to come to the black board.

Tess: I estimated how much the red part was (refers to the slide bar) when you take a look at the whole slide bar. I did it this way: I took half of it, and then half of this half, and then again half. I came to 12,5 % and, well, that's how I did it.

Teacher: It's not completely clear to me. Can you show us by doing it?

Tess: (draws a bar on the blackboard) I drew the red part in it first, it was about this. Then I took half of the whole bar, is 50 %, and half of it again is 25 %, and half of that again. So the red part is 12,5 %, 25 divided by two.

Tess estimated the red part of the slide bar at 12,5 %, which isn't that bad. However, this estimation does not help her to solve the question about how many hours the battery can work. Another student is allowed to help.

Walter: Well, if it's 12,5 %, then that is almost the same as 10% (indicating the red part), and that's easier to work with. So I took 10 % of 20 hours and that should be two hours.

Tess and Lisa solve the next battery task together again. The task is: If a full battery lasts 12 hours in a walkman, then how many hours can Marije still listen to her walkman? (The battery is indicated to be filled for 60%). They talk about it and try to figure it out.

Lisa: So this [battery] is 12 hours.

Tess: …50%, anyway 6 hours listening, you can at least listen for 6 more hours.

Lisa: … (silent)

Tess: Because this is 12 hours, look, this is 6 hours and then she can listen a little more.

Lisa: Yes, 6 hours, but it is 60 % not 50 %. 6 hours and a quarter perhaps…

Tess: I guess it's 7 hours. Yes, watch this: you have to divide this battery by twelve. 1, 2… wait a minute

Lisa: Do this… yes, let's draw it. You know what, we'll do it this way. Like this (she draws a simple bar)

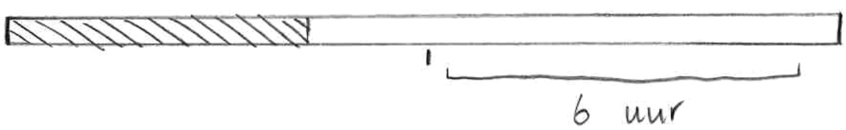

Figure 9: Model of the battery at a more formal level, designed by Lisa and Tess. Below the right part of the bar they wrote: '6 hours'.

Lisa draws a simple, thin bar, which is a model that does not refer to the concrete context directly. It seems that Lisa moves from her earlier models at 'referential level' to a model that is at 'general level' (see Figure 9). Our interpretation here, is that Lisa is inspired by (1) the extra task the teacher introduced earlier with the slide bar, and (2) Tess's idea to draw a bar at the black board to explain her thinking.

Lisa: Lets take this bar as a model and divide it by ...

Tess: No, you cannot compute it exactly with that drawing. Here, use a ruler, it will be much easier then.

Lisa: Well, let's see. This shaded part is 4 centimeters, this whole bar is 11… (She draws a new bar with her ruler). Ok, so this was half, 6 hours wasn't it? […] and now I still have a small piece left.

Tess: That little piece should be about an hour, look!

Lisa: Are you sure?

Tess: Watch this, Lisa. It is still not exact… This whole bar should represent 12 hours, but we made it only 11 centimeters, 10 pieces on it, so each piece is 1 hour, or a little bit more…

Lisa: But stop… No, … yes, one centimeter is a little bit more than one hour, so we did it all right. We have a piece left, next to the 6 hours. It should be about 6 hours, perhaps 6 hours and 3 quarters, or just 7 hours, yes, it should be 7 hours.

Although the answer is not exact, it is all right for practical reasons and it shows clearly that Lisa and Tess understood the assignment. The students then talk about the task in a whole-class discussion under the teacher's guidance. To show that a battery is filled for 80%, the teacher draws sort of a ruler. He saw this model in students' work. He divides this ruler, in fact it is more like a thick number line, in 10 pieces and practices with the students the use of this model. This model is not context-specific; it can also be used in other situations.

In the next battery task the students are asked to design a representation which shows how full the batteries of the radio and the pocket-lantern are. Once again, this task could be solved by several kinds of models. An example of Lisa's and Tess's efforts is shown in Figure 10.

In their model, the concrete reference to the battery is gone. They draw bars and shade part of it. They use the whole width of the rectangle that is given to make the drawings in (see Figure 10).

Lisa: (referring to the radio battery) 20 %, well, that's 1/5th, isn't it?

Tess: (referring to the pocket-lantern battery) and 2 hours left.

Lisa: Huh? You're too fast. I don't understand!

Tess: The battery could work for 20 hours, but now only two hours are left. So in our model we should draw the 18 hours that he had worked, and the two hours left. If I make this bar, then half of it is 10 [hours], and half of that is 5 [hours], and half of that again is 2,5 [hours], so then this should be about two hours.

Lisa: And this bar dividing in 5, then we have about this left.

Tess: So, this is 20 %, and this is 2 hours. This piece of the bar is bigger, so 20 % is more!

Studying Tess's and Lisa's models in this paragraph, it can be noted that their models are increasingly less detailed and less situation-bound than the models they started with. The first model in the battery tasks could even be said to be at situational model, given the fact that the students had the opportunity to manipulate the battery themselves, and their exact duplicate of the battery.

Figure 10: Models of the 'radio or pocket-lantern' task. This task says: 'Radio or pocket-lantern? In which one does the battery last longer?' Both batteries have a lifespan of 20 hours. The person left states: 'My battery lasts for another 20%'. The person at the right says: 'My battery lasts another 2 hours.' The model below is designed by Tess. In the model at the left she wrote the legend 'black is empty' and 'white is full'. The same legend is used in the model at the right.

To see the change, for example take a look again at Figure 8 and compare it with Figure 9 and 10. The solutions that were discussed and showed on the blackboard earlier, made them decide to make a shift from concrete battery drawings to bar models. The context is becoming less important than the characteristics of the mathematics. In the last task, they seem to use the principle of the number line. In this particular series of tasks about batteries, they are no longer working at *'referential level'* but on *'general level'.* It is interesting to speculate about the reasons for these transitions. We believe that at least three plausible reasons may need further reflection:

1. *Nature of variables involved:* the tasks in the lessons make use of different sorts of quantities. The coffee example uses coffee as entity, whereas the others use money/rent and time/energy. In the first case we have a concrete variable to be modelled, while in the two latter cases it is a more abstract, continuous variable to be modelled. It might be that the representation of concrete things is prone to be referential while the representation of an abstract continuous variable is more evidently expressed as an easily generalizable drawing. What, for that matter, could be the characteristics of time/energy (or money/rent) which should be expressed in a drawing?

2. *Nature of involvement:* another difference between the different task situations concerns the structure of the tasks themselves. The coffee and money task were presented as paper-and-pencil tasks, probably not directly connected to the imaginations of the pupils. The battery task, on the other hand, started with a concrete presentation of the battery and its inscripted measure of charge, which already looked like a model itself. The battery task starts off, so to say, from a situational model strongly suggesting a focus on a line representation of charge. The latter task probably enhances the involvement of the pupils in the task and directly suggests imaginations of how to represent this abstract quality of batteries.
3. *Exigencies of the communication process:* a third possibility is in fact a consequence of the previously mentioned reasons. Both the abstract nature of the variables and the involvement of the pupils with the real things caused an attitude in pupils to be as precise as possible in the representation of something (energy, rent) that is not easy to represent for these pupils. The course of the discussions between Tess and Lisa shows that they use their drawings as tools for solving a communicative problem, as can be seen in the episode mentioned above starting with "Are you sure?" and Tess answering "Watch, Lisa, It still is not exact.." Observational studies in young children showed that both the "Are you sure?"-question and the exigencies of communication processes stimulate pupils to reflect on precise representations that leave out those things that don't seem to be essential for the communicated message itself (see van Oers, 1994; 1996). Although it is speculative to say, we assume that the need for further articulation of the essential as well as the cutting out of those elements that don't seem to be helpful for the communication are triggered by the pupils' wish to solve communicative problems as clearly as possible. The gradual transition to models on a general level may be the result of this same process.

We can see the same kind of communicative dynamics at work in the following example. Here again the exigencies of clear communication forces pupils to be precise and parsimonious in their representations.

Figure 11: Model of 10 percent question, designed by Sam.

General Level, or Model-for

The next lesson, the teacher starts with a general question, to provoke an abstract model. He, then, applies this model in a task, to show how one could use it (see Figure 11).

Teacher: Who can show me what 10 % is?

Sam: How? With money?

Teacher: No, I just ask, can you show me what 10 % is?

Sam: (comes to the black board) Eh…, I draw a line, and I divide that line in 10 pieces. And then I draw an arch below the first piece, and I write $1/10^{th}$ there, and 10 %.

Teacher: Ok, what do the others think of this model?

Mandy: You have not shown how much the whole line is.

Teacher: How can you improve that?

Lisa: By writing it on the right side of the line: 100 %.

Teacher: Can we use Sam's model to solve a task? Say, a CD costs 50 guilders, but now the shop celebrates a jubilee and you'll get a 10 % discount.

Ethan: Well, 10 % is one piece of the line Sam drew, so you have to divide 50 guilders by 10, and then you know how much discount you get.

Teacher: Let's write it down in Sam's model. Sam put the percentages at the bottom of his model, so I write the amount of guilders on top of the model. The whole line is …?

Kids: 100 %.

Teacher: And that's how much in guilders?

Kids: 50 guilders.

Teacher: So, if this whole line is 50 guilders, how much will this little piece be?

Kids: $1/10^{th}$, 5 guilders, 10 %.

Teacher: Yes, very good! I hear several answers. This little piece of the model stands for $1/10^{th}$, but also for 5 guilders, and also for 10 %.

Then, students are asked to represent the various sports children of a group practice. 35 % of the children play soccer, 15 % of the children play volleyball, and so on. The teacher first asks the students to think about ways to represent the distribution, and then invites Tess to come to the black board and show her solution (see Figure 12).

Tess: Well, I do it this way. You draw for example a class here. And then you shade the amount of 35 %, and you write soccer in it.

Teacher: But wait, I don't understand. How do you draw a class?

Tess: Let's pretend this is a class (she draws a rectangle at the black board).

Teacher: Aha. You draw something that represents the class, but it does not really look like a class?

Tess: Yes. And I know that this is 50 % (draws a vertical line in the middle, which divides the rectangle in two equal parts). Then this is 25 % (by a horizontal line she divides one of the '50%' parts in equal halves), and this should be about 35 % (draws a line a couple of centimeters below the '25 % line').

Teacher: Tom, what is Tess doing? Can you explain?

Tom: She first took half, and then she took half of it again, which makes 25 %. And then you can estimate where 35 % would be.

Teacher: OK, let's fill in the next: 20 % of the children play volleyball.

Tess: Well, here I have 15 % left (points below her shaded 35 %), and then I need 5 % more … Eh… but that is a little bit unclear…

Teacher: Who has an idea to make it clearer?

Marc: Why not make all vertical strips, instead of horizontal and vertical lines? So that one vertical strip is 10 %, and 2 strips are 20 % and so on. At the left is 0 % and on the right is 100 %.

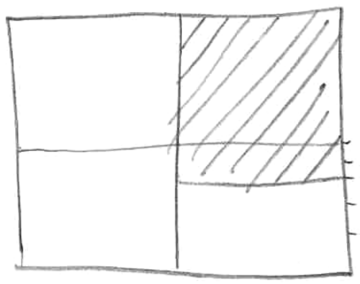

Figure 12: Model of the class assignment, designed by Tess.

Here, the model Tess thought of is used as a means to solve the question. Other students are involved in the process when the teacher asks another student to explain what Tess is doing. However, while working with it, Tess remarks that it is perhaps not that clear. The teacher uses this opportunity to ask for ways to make the model clearer. Other students have ideas, and the model is improved.

Making a model of the amount of marbles several children own is the next task. Here, a child named Jim has designed a model, which shows marble bags with each 5 marbles in it. The students are invited to think of a better model. Lisa designs a model like Jim's, but completes it with a legend that indicates that one marble stands for 5 marbles. Dave designs a model with a list of names on the left, and a tallies system on the right. After Lisa and Dave showed their solutions at the black board, the teacher asks the students if they think these new models will still be helpful when amounts of more than 100 marbles have to be represented. Both Dave and Lisa answer in the negative. Marla proposes to write the names on a list, and then write the number of marbles behind the names. The teacher asks if it is then in one quick view visible who owns the most marbles. Marla nods, but Roy disagrees. He suggests making squares, and each square should indicate 10 marbles. Putting all these squares in a row behind the name of a person will show in one view who owns the most marbles. In fact, Roy designs here a histogram, inspired by the ideas of his fellow students (see Figure 13). He also improves his own model, a model with all little squares, where one square indicated one marble.

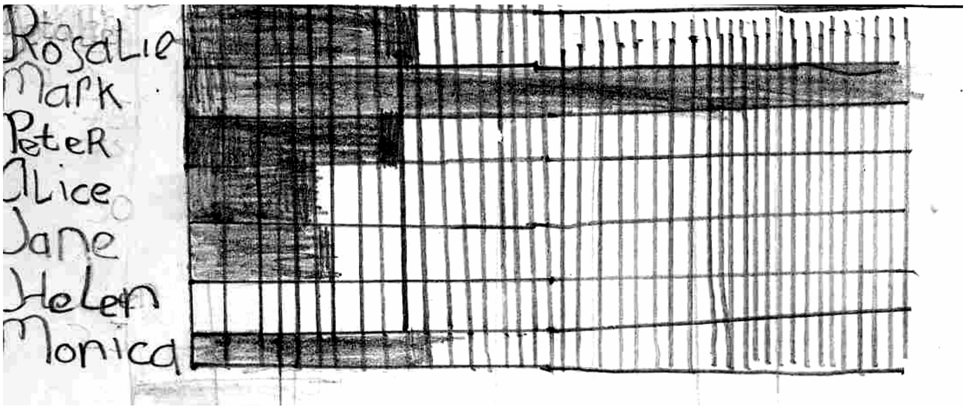

Figure 13: Model of the marbles assignment, designed by Roy.

At this point of the lessons, the teacher decides it's time to reflect on the designed models thus far. In a short session, the children are invited to name the models they thought of.

Teacher: Well, you've all seen a lot of different models we designed until now. What models did you remember?

Kids: Circles, puppets, bars, histograms, number line.

Teacher: Which of these models was the least useful for you?

Mandy: I did not like the puppets. They were not easy to read, and it takes you a lot of effort to draw them.

Teacher: And what models do you like best?

Kids: Circles, bars, number line.

Teacher: Why the bar, Kim?

Kim: It almost always works. And it is easy to draw.

A new task handles a running tournament. The students read a story about two schools, who send their best girl to run a two-kilometer race. The race is described in detail, and the students are asked to design a model that shows how the race develops in time. It should be possible to see at which distance each girl runs at a certain time. Let's see what Lisa and Tess talk about.

Tess: Which color has Jessica's shirt? Ah, blue.

Lisa: How shall we do it?

Tess: I'm going to draw a tube.

Lisa: Yes, and it should be 10 cm, because the race takes about 10 minutes.

Tess: And then we'll shade blue where Jessica is at 3 minutes or so.

Lisa: Here, this is the start. And she has run 0 meters.

Tess: And here the finish. That's 2,000 meter, wasn't it?

Lisa: (divides the tube by 20 small lines, and counts up to the tenth line) This is 1,000 meter.

Tess: (reading the text) So half the race costs her 5 minutes.

Lisa: Let’s write that here, above the tube.

Tess: And she does a quarter of the race in 2,5 minutes… 10 minutes is the whole race, 5 minutes is half, and 2,5 minutes is a quarter.

Lisa: But here it says that she is at the 600-meter point in 3 minutes. Here is the 600-meter point. Yes, that should be in 3 minutes!

Tess: We can shade that part blue (she colors the tube until the 600 meter point completely blue).

Lisa: And after 5 minutes she is at the 1,000-meter point. Let’s shade that part in another way.

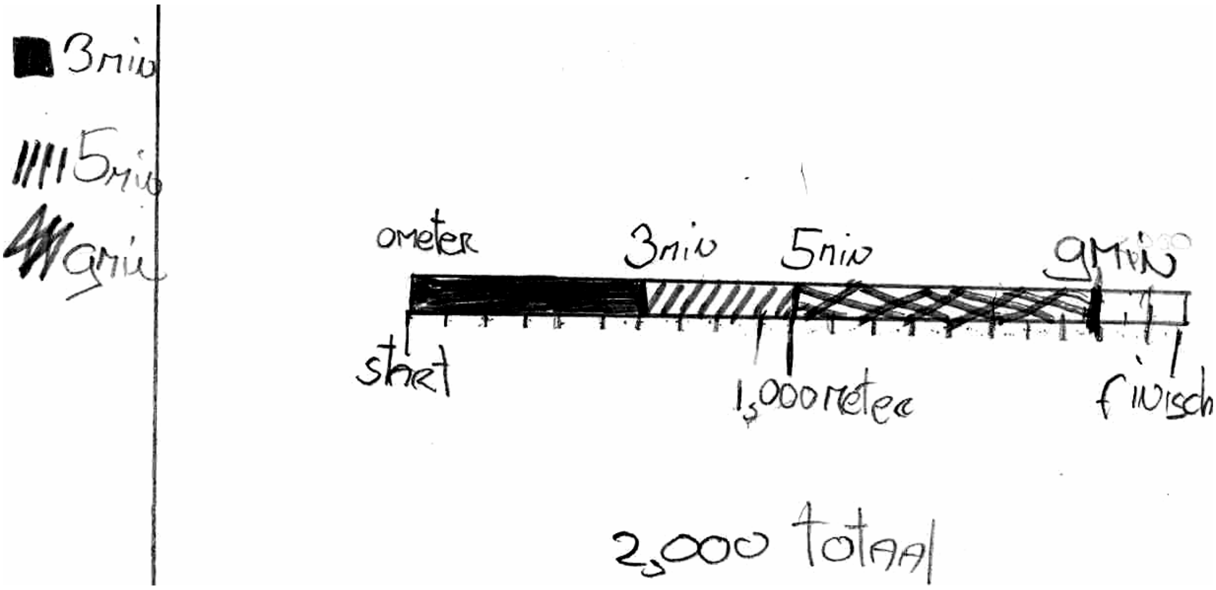

Figure 14: Model of the running race, designed by Tess and Lisa. Above the bar it says: ‘0 meter, 3 minutes, 5 minutes, 9 minutes’. Below the bar it says: ‘start, 1,000 meter, finish, 2,000 total’.

Here, Lisa and Tess use a model, which is not completely tied to the situation (see Figure 14). This tube could be used in various other situations. They have used something like that before, but called it a bar then. This time it looks like a double number line, the distance can be written at one side of the tube, and the minutes at the other side. However, Tess and Lisa are not very consistent yet. They use time and distance at both sides of the tube. Other children also used bars like Tess and Lisa. The teacher notices that distance and time are not separated on their models too. He therefore asks one of the students to draw the model at the black board and share it with the others. Then, he explains that it is not clear when you use several measurements intertwined. He proposes to use one side for time and the other side for distance. The students agree.

In the last task (show it in percentages: ‘3 out of 5 people use a trajectory ticket’), Tess and Lisa use a circle diagram for several tasks. This is not immediately the case; they start with a less formal model but switch to circle diagrams (see Figure 15).

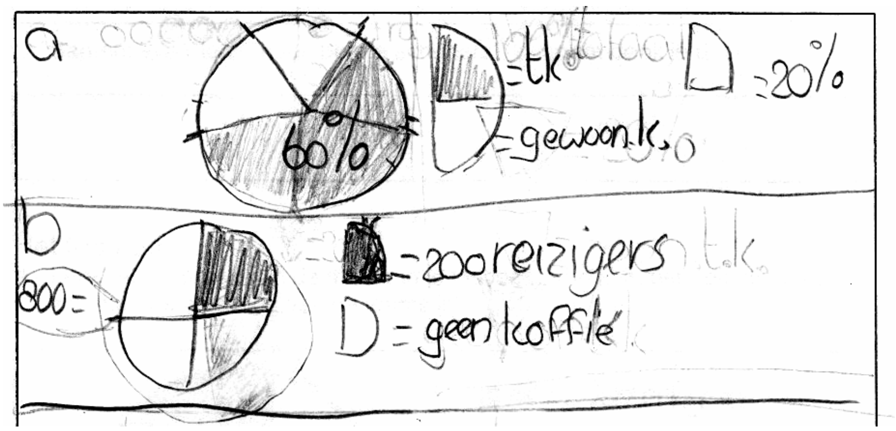

Figure 15: Model of the train travel task, designed by Tess and Lisa. Task A has a legend which says: 'shaded is traveling with a trajectory ticket, and white is a normal train ticket'. Task B's legend says: 'one shaded part is 200 passengers, and white means no coffee'.

Lisa: I draw 5 balls, and then I shade 3 of them…. But how many percent is it then?

Tess: … (silent)

Lisa: Oh, why are we doing that difficult? Why don't we use such a circle?

Tess: Yes, good idea!

Lisa: How does it work again? Oh yes, divide it by 5. Like this.

Tess: That's great, how you divide it by 5. That's not easy…

Lisa: Here, this is 1, 2, 3, … look Tess!

Tess: Then, one piece should be…

Lisa: 20 %. How much pieces should we color?

Tess: Eh, 3 out of 5. So the answer should be 60 %.

Teacher: Class, I asked Kim to show us her model. Show us, Kim.

Kim: (draws a bar, divided in 5 equal pieces, and with a legend that one piece means 20 %)

Teacher: How did you think of that?

Kim: Well, I know that 100 % is all, and 100 divided by 5 is 20 %. And the assignment is 3 out of 5 people, so I have to shade 3 pieces, is 60 %.

Lisa: (whispering to Tess) That's smart too!

In this last paragraph it seems that Tess and Lisa more and more design models that are at a higher level. Concrete references to the situation given are sometimes visible (like Lisa's marble solution), but most of the time they leave out the details. Inspired by each other, inspired by the models their fellow students develop and inspired by a need for clear communication, they grow to a more general level of representing tasks. The models they develop are now most of the time useful in other situations and tasks too. Furthermore they show they use these formal models in several occasions.

Interestingly, the need for clear and unequivocal communication drives the pupils into modeling that confines itself to the things that are taken to be essential. Moreover, the examples show how reasoning with the generalized model of the situation produces new information (re-descriptions) of the situation that can only be deduced from the model, but which cannot be seen in direct perception (like the notion of percentage). So the generalized model indeed helps pupils to go beyond the information given.

It is clear that the process is also stimulated by the comments of the other pupils in the classroom. In the course of the lessons this thinking in general models for the solution of problems has become a property of the classroom communities' discursive activities. The example shows that developments at the individual level (Tess and Lisa) co-develops with developments at the level of the classroom community. This finding is consistent with Cobb's (see Cobb, Wood & Yackel, 1993) theory about the reflexive relationship of development of the individual and the sociocultural evolution of a collective culture. The transition from concrete models *of* to generalized models *for* is most likely in a community that ask for certainty and clarity.

Studying the models used by Lisa and Tess in the series of tasks, you could say there is a development: from a *'model-of'* to a *'model-for'*. Of course, the question remains if this development is permanent. In fact, we believe this development to be an reiterating process: with new tasks children tend to fall back in earlier forms of representation, and from there they build their understanding up to a more formal level. This process will develop quicker when students get more experienced. As was already mentioned, the learning gains of both Lisa and Tess were above average, compared to other students.

Conclusion and Discussion

The case study we describe in this article is embedded in a larger study, as mentioned in the Introduction section. Our research question was: What are process characteristics of the emergence of formal models in primary mathematics education, under the conditions of co-constructive elaboration of students' informal models? We operationalized the process characteristics in terms of shifts in model use. In the Theoretical Background section, we discussed levels of model use, initiated by researchers as van Hiele, Freudenthal, and Gravemeijer. We took Gravemeijers' levels of activity as theoretical framework to search in episodes of lessons for developments in students' model use. Gravemeijer describes four levels of activity: situational level, referential level, general level and formal level. It appeared that the situational level could hardly be found, and the formal level could not be found in the episodes. Most of the students passed the situational level already at the start of the intervention. However, in our view the most interesting shift in levels between referential (model-of) and general (model-for) level, is amply available: the models shift from strongly context-bound models to models that are applicable in several situations. Furthermore, examples of hypothetical thinking, which in our view is necessary for the ability to create models-for, can be found in the interaction.

The developments, or transitions in model use, were sometimes well visible, but sometimes less explicit. Developments became visible in students paper work, class-wide discussions, discussions between children in pairs, or when they shared their ideas on the black board. We could see that some students started their problem solving already with models at general level, while other students used models at referential level. Each student had the possibility to start at his or her own level.

At first, the two students we followed closely made models that referred directly to the concrete situation in the given problems. Initially, the representations could be placed at the referential level and are 'models-of'. The models they create cannot be used to solve other mathematical problems besides the one for which the model is made. Some other students already show models at a less informal level, like a bar model. These models can be seen as the first signs of 'models-for'. Clearly, differences between students already exist at the start of the lessons.

The assignments in which the students have to work with the battery contexts show a development in the models of Tess and Lisa. At first, they draw an exact copy of the battery as described in the problem, even with the exact mentioned colors. Later on, they design models that are not exact replicates of the batteries, they draw a bar or a line as a model to solve the problems. From then on, they use in other tasks models they developed earlier, or models they learned from other students. Their models seem to become more at a general level, without concrete references to the situation in the tasks.

In the verbal interaction between Tess and Lisa, it sometimes becomes obvious that they make progress, and jump from one level to another. For example, in the last task Lisa sighs: 'Oh, why are we doing that difficult? Why don't we use such a circle?', indicating that this formal model at a general level is more useful for her now. They help each other where necessary, explain their ideas to each other and to the group when they are invited to do that by the teacher.

We can also find examples of hypothetical thinking in episodes. Think for example of the task that Tess explains on the black board, where the teacher asks her how she wants to represent a class. Tess here replies: 'Let's pretend this is a class', while she draws a rectangle. This hypothetical thinking is typical for working at the model-for level.

As stated earlier by Lehrer and Schauble in the Theoretical Framework section, the informal models, together with experiences with the situation, will support the mapping of reality. After some experiences with such modeling, students will learn that the resemblance is less fundamental than the functional representation, and will be prepared to work with a kind of models that do no longer stick to similarities between the model and the real world. This is exactly the development that became visible in our examples.

How can we explain this development in models? Two factors are important here. First, the students in the designing group have explicitly been taught in the process of model designing. And second, we believe that the communication processes that occur between cooperating children urges them to sharpen their answers. Children have to explain their beliefs, understandings and models, in order to show the other person their thoughts. In this process they feel the need to develop their models even more, as a means of communication. This finding is consistent with Cobb's theory (Cobb et al., 1993) about the reflexive relationship of development of the individual and the sociocultural evolution of a collective culture. The transition from concrete models *of* to generalized models *for* is most likely in a community that asks for certainty and clarity. In studies by van Oers (1996[b]) is shown that children only feel the need to develop models, when there is an explicit communicative purpose (co-operation, for example).

Can we draw the general conclusion that there is a development from a model-of to a model-for in the models the students designed as a consequence of the intervention which was directed to learn students how to design their own models? On the basis of only two students, and a couple of other students who participated in the process of developing models, we cannot generalize the findings.

Fortunately, we are able to embed our findings from these cases into the context of the study as a whole. In an earlier case study we described the construction process of models, by following two students in-depth, one from each condition. We then came to the conclusion that the student in the experimental condition was better able to use her self-designed models, in comparison to the model use of her counterpart in the providing condition. Results in our other, more quantitative publications also point in that direction. Students in the experimental, designing condition seem to perform better on knowledge tests, as well as on a transfer test than their counterparts in the control condition (van Dijk et al., in press[b]; van Dijk et al., 2001). Taken together (1) our process analysis in the case studies and (2) the quantitative outcomes for all students in both conditions, we have clear evidence for the effectivity of our approach in terms of learning results as compared with the control group. For practical reasons, in this article we concentrated on the processes that occurred during the intervention.

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6. Conclusion and Discussion

Introduction, Research Question and Hypothesis

Modelling in primary mathematics education, and its effects on learning outcomes is the overarching theme of this thesis. More particularly, we studied the co-construction of models for use in complex problems. Several studies on the topic of modelling in mathematics education were conducted in order to examine the effects on learning outcomes of two programmes to model use. At the start of this study, the following research question was formulated:

> What are the effects of an experimental learning-in-context programme – in which learners are seen as designers – on the learning processes and learning outcomes of pupils in primary mathematics education, compared to the learning outcomes of pupils in a control group in which ready-made models are provided by the teacher?

It was expected that pupils in the experimental programme may show better results than pupils in the control programme. Learning to design models may lead to deeper insight into principles or conceptual structures that lie behind the tasks. Such deeper insight will facilitate learning. Moreover, it was expected that differences would be found in the transfer of learning in new and complex situations. Due to the active involvement of the pupils and the fostering of insight in a constructive approach to problems via model construction it was expected that pupils from the 'designing condition' would be better equipped to solve complex problems.

We studied this research question from a socio-cultural and a mathematical-didactic perspective. These perspectives imply the assumption that children learn mathematics by participating in mathematical activities either in school or in everyday practice, and by reflecting on, discussing and constructing signs and meanings (semiotic activity) with others who are more knowledgeable, including peers and teachers. The perspectives also imply that children's contributions are taken as the starting point, and that children themselves are seen as knowledge constructors. These aspects were integrated into the associated research project.

In the following paragraphs the results of these studies will be summarised, linking the several chapters of this thesis back to the main questions. In addition, the limitations of the present study will come under scrutiny. Finally, the educational implications and theoretical gains of the results will be discussed, and directions for future research will be outlined.

Summary

When children progress in their school curriculum they increasingly encounter mathematical problems where modelling the situation could be a helpful strategy in representing the problem situation and in coping with the complexities. In the context of the research questions a research project was undertaken which attempted to examine the feasibility and effects of a designing programme for mathematics learning in the upper grades of primary school. In a pretest–posttest control group design two programmes were systematically compared. The first concerned pupils who were involved in a socio-semiotic activity of maths model construction (designing condition, experimental group); the second concerned pupils who were provided with ready-made models constructed by others (the providing condition, control group). It was assumed that the control group children would get minimal experience with designing activities and would consequently learn less from their lessons than the pupils from the experimental group. It was also expected that differences between the experimental and control group would become especially manifest if the pupils had to solve complex problems on a transfer test. To answer these questions, several studies were conducted. First, a small-scale case study in two classrooms was carried out. This was followed up with a field experiment involving ten classes, in which 238 pupils were studied for a period of three weeks. Both studies will be reported in this section.

In the case study, a ‘providing’ and a ‘designing’ classroom were contrasted. Chapter 2 is devoted entirely to this case study. Two pupils, one from each classroom, were selected. They were followed closely in their learning processes (Van Dijk, van Oers, & Terwel, in press[a]), focusing on the construction processes that occurred in a series of lessons involving problem-oriented percentages tasks. Video footages uncovered several differences in the approaches of the two pupils. Though we have to be cautious with generalising from a case study, ‘designing pupils’ appeared to be more confident with their self-designed models than the ‘providing pupils’ with their provided models. This case study clearly showed the intended activities of teachers and pupils. In addition, some indications were found that the expected learning results had occurred. It seems therefore reasonable to assume that children in the upper grades of primary school are capable of designing models in co-construction. This qualitative study yielded a firm foundation for the next study and provided a convincing case that the intended processes can be realised in normal classroom settings.

The experience and knowledge gained in the explorative case study was then applied to the field experiment (Van Dijk, van Oers, Terwel, & van den Eeden, in press[b]). This study was mapped out in two separate analyses. The first analysis (Chapter 3) dealt with the outcomes on the posttest, and the second analysis (Chapter 4) with the outcomes on the transfer test. 10 classes, 10 teachers and 238 grade-5 pupils (age 10-11 years) were involved in this field experiment, with an experimental group and a control group. 117 pupils were assigned to the providing condition. This model-providing programme is the way in which regular education takes place in most primary schools in the Netherlands. The experimental (‘designing’) group, consisting of 121 pupils, was exposed to the same mathematical content, but here the emphasis was on the ‘guided co-construction’ of models by pupils and teachers. The study was designed to examine whether the tentative conclusions arrived at as a result of the case study also hold for other topics, and a larger group of pupils. This time, we focused on the quantifiable learning outcomes.

This larger field experiment showed that pupils who learned to construct models in the experimental programme scored significantly better on both the posttest and the transfer test than children who learned to work with models provided by the teacher. The conclusion is therefore warranted that learning how to design models in the context of a strategy for solving mathematical problems in daily-life situations is a valuable approach, and puts pupils in a better position as compared to pupils who are provided with ready-made models. In a multilevel analysis we searched for differential effects for low and high achieving pupils. This analysis showed no differential effects, thus indicating that all pupils of the experimental group benefited from the programme.

In order to advance our theoretical understanding and to be able to improve this programme even more for practical use in the future, we had to learn more about the way model learning in pupils proceeds. In the quantitative analyses of the data little attention was paid to the process of learning to construct models, or to the emerging models. This aspect was elaborated in further detail in a qualitative observation (Chapter 5) by analysis of videotaped lessons and pupil materials by a few pupils. In the analysis Gravemeijers' theory of levels of activity (1997 [a]) was applied. Passing through these four levels, students were involved in a process of generalisation. For our study, the most interesting part of Gravemeijers' theoretical framework is the shift from the referential level to the general level, or from 'model-of' to 'model-for'. In this transition, the initial context-specific model is transformed when it is generalised over situations, and assumes an identity of its own. This newly emerged type of model can function as a tool for mathematical reasoning on a formal level. Several examples of this shift were found when developments in the model designs of two pupils were followed in depth. For example, one pupil, who designed a model for a task about 'coffee left in a pot', continued to use this model in a task about the entirely different subject of rent. Although the situation in the task was completely different, she reapplied the model and adapted it to the new task. Examples of this transition between levels occurred more than once, indicating that pupils can make the shift by generalising models they designed earlier.

The aim of this qualitative Chapter 5 was to learn more about the process that characterises model learning. By linking up findings from these qualitative analyses with the outcomes of the quantitative analyses, Chapters 3 and 4, it seemed reasonable to conclude that pupils who progressed in the direction of models-for also seemed to be the ones who coped relatively well with the transfer tasks. Children who raise levels in their working with models are better able to accomplish transfer.

Taken all together, the results mapped out in the Chapters 2 to 5 clearly show the positive effects of the experimental programme. Experimental pupils clearly outperformed the control pupils on both the posttest and the transfer test with effect sizes of respectively .40 and .63. Our theoretical point of departure and the results of our experiment make us believe that designing models in co-construction may lead to deeper insight into the meaning and use of models, and consequently make possible a more flexible approach to problem solving. But what do these results mean for educational practice?

<u>Theoretical Gains and Educational Implications</u>

Which conditions have to be met in order to be able to effectively apply modelling in mathematics education? Several factors would seem to be important:

(1) Modelling can be seen as a form of *semiotic activity* and as a *process of mathematizing*, and manifests itself particularly as a continuous process of improvement in the activity of representing. For educational practice this means that pupils should get the opportunity to reflect on signs and meanings, for examples models. Exchanges and discussions of children-designed models can facilitate the

improvement of models. Some promising models are selected and discussed in class. These models are then elaborated jointly.

(2) As modelling is basically a process of constructing tools that must fit the requirements of a situation as well as the demands of personal interests -- increasingly complying with external constraints that follow from the need for communication -- the process of modelling must be founded on *active personal involvement*. Active involvement (as took place in the designing activity in the experimental condition) is necessary to acquire the skill of modelling. We interpret the active personal involvement as a condition that calls for a constructive mode of learning. Offering open problems, which can have several different solutions and tags, can evoke the active involvement of pupils in mathematics. In this way, every individual child is taken into account, and each child is actively involved in the process of problem solving. Furthermore, children can raise their level of mathematics as a result of being confronted with the ideas of others. Teachers can organise such confrontations in several ways: by discussing the various models on the black board; by letting children walk through the class, showing their models to each other and discussing them in a informal way; or by asking children to present their reflections, developments and modelling work to the class as a whole.

(3) In order to promote cultural quality in modelling, participation of an adult or/and a more knowledgeable peer is necessary. From our research evidence we may conclude that culturally relevant models are often the outcome of a *co-constructed process*. Co-construction is likely to contribute to the quality and flexibility of the produced models as socio-cultural experience can be inserted into the construction process and the presence of multiple points of view calls for flexibility and intersubjectivity. Again, exchange of ideas is important here. The participation of an adult (e.g. a teacher), who can assess the value of models directly, may take the use of models to a higher level.

(4) In modelling there is a *progression of representing* from an iconic (pictorial) mode (the products of which are usually called *models - of)* towards a symbolic mode that manifests the ability of going beyond the information given and deriving implications for the understanding of the modelled entity (object, situation, action), the products of which might be called *models-for*. Paying explicit attention to models-of can evoke this progression in representing mathematical situations. Models-of are often imprecise, difficult to draw, and unstructured. Children should learn to recognise these less desirable properties, so that they can improve their models by adapting them.

It is important for method developers to focus more on children's skills in modelling situations. An increased number of open complex problems, requiring a designing-approach, should be systematically added to maths textbooks, without ready-made models being offered directly. A less prominent role should be reserved for ready-made models, at least in situations where pupils are not given the chance to first think of and design their own models.

To sum up, using a designing-programme in education crucially requires a focus on activities in which children can add meaning to the tasks by active personal involvement. This approach should be a co-constructed process, in which adults and/or peers participate and in which special attention is devoted to progression in representations from an iconic mode to a symbolic mode.

Apart from additions in experimental knowledge, the research on which this thesis is based also yielded some new theoretical insights. As mentioned in the introduction and several other places, researchers such as Mayer (1989), Perkins and Unger (1999), Rosenshine, Meister and Chapman (1996), and Hattie, Biggs and Purdie (1996) all asked themselves the following question: How can pupils obtain support from being familiar with strategies that lead to greater insight into principles, conceptual structures or theoretical models? In this context, Rosenshine et al. raised an important question. Is it better to provide pupils with models and strategies, or should pupils learn to generate these models or strategies themselves? However, an approach that was even more interesting in our view was not mentioned in Rosenshine et al.'s study. This led us to the strategy of leaving the contrary pair 'providing versus generating' aside, and instead to focus on a new contrary pair: 'providing versus designing in co-construction'. Together with the robust results in learning outcomes of the designing programme presented here, the addition of this new contrary pair is offered as a contribution to the discussion on 'providing versus generating'.

Limitations and Future Research

Looking back on this project, some aspects might have been dealt with differently. Let us discuss some of the issues involved here:

1. This study involved making interventions in the subjects of percentages and graphs. Time limitations caused us to focus primarily on percentages. The effects on the learning of graphs with a designing-approach have not yet been reported.
2. Lack of time also forced us to carry out only limited qualitative analyses. The video footages and protocols of lessons still contain a great deal of information, for example data about the role of the teacher and the role of other children in the learning processes group. Analysis of the video footage proved to be extremely labour intensive and time-consuming. In this study we only had the opportunity to follow two children closely during the three weeks of lessons, and to reflect on their learning processes. It would have been even more interesting if we could have followed several pupils of mixed mathematical abilities, and from classes in both conditions, with a view to generalising the results of this study.
3. How broad will be the bandwidth of ability in order to be able to benefit from a designing approach? Are there differential effects for high and low achieving pupils? In this study we did not found differential effects. Both high and low achieving pupils seem to benefit from the experimental programme. However, our sample (10 schools) was just big enough to justify the use of a multilevel analysis. It is possible that a larger sample of schools could show differential effects. More research into possible differential effects for high and low achieving pupils is needed.
4. A three-week intervention is very short. It would have been desirable and interesting to follow the developments of the ten classes over a longer period of time, focusing on retention of results, for example six months after the end of the intervention.

Reading this thesis, the reader might get the impression that letting pupils work with ready-made models is in all cases less valuable than letting them work with models they design themselves. However, a possible advantage of providing ready-made models is that they are developed by experts (e.g. teacher or curriculum designer), who are able to design models with 'future possibilities'. Such 'conceptual models' are designed with an eye on further introduction into scientific thinking and can be used in various situations and topics within the domain. However, although this kind of representations may support understanding, they do not warrant real understanding. It has been shown that not all children can see equally well through ready-made models (see Lampert, 1989; van Dijk, van Oers, & Terwel, in press). In that case, the mathematical concepts embodied in the representations are only there for experts who already have those concepts available (Gravemeijer, 1997[b]). For students there is nothing more than the representation itself, but it has no personal meaning for them. As a result, the model cannot serve as a means to overcome the gap from concrete to abstract, and cannot fulfill the bridging function. Real understanding is facilitated by active personal involvement; pupils add meaning to the tasks, and may appropriate these tasks as their own problems. They learn more by being more actively involved. There is presumably less of this kind of personal involvement when models are imposed.

In the designing programme a ready-made model can still be helpful in that it confronts pupils with the ideas of others, or, in the case of blind alleys in the designing process, the teacher can decide to present a model himself. In such cases the difference between the designing programme and the providing programme is that these ready-made models only come across when the children already thought of constructions of their own. The ready-made models are not starting points, as is often the case in the providing condition.

For practical reasons, this study closely followed only a couple of pupils in their learning process, and these pupils were all average mathematics students. Consequently, the question of providing versus designing remains unsolved with regard to pupils who either achieve far above, or far below the average in mathematics. As shown by the quantitative results of the multilevel analyses, no differential effects were found, indicating that these pupils also clearly benefit from the designing programme. But what kinds of processes occur in these pupils with regard to model development and level raising? Do these developments differ between high or low achieving pupils? No answers to these question are forthcoming in this thesis. In this respect, too, further research will be needed.

If this designing programme is to be considered really important in classroom practices, a more systematic focus on it would be desirable. From a designing-perspective children should learn to model from a young age on. Such modelling could be done in playful ways that help children to see the need for the designing approach. A longitudinal study is required in which children are followed for some years, from the early schematising process into modelling that is constrained by certain disciplinary rules. Fortunately, our department has made a first step in this direction by authorising a new research project, which has just started. This research project will address the question of whether a one-year involvement in symbolic representational activities by 5-year-old children (drawing, making and using diagrams, maps, and schemes) in different contexts has a positive effect on processes and learning results during the subsequent introduction of school mathematics, a year later.

There is also a need for new design experiments that start from the outcomes of the underlying research and delve more deeply into the problem of the co-development of the classroom community and individual learners. Co-construction not only enhances the learning of individual learners, but also spins off into the learning community that they are part of. If this line of reasoning is correct, we have arrived at the point where the curriculum stops being a pre-designed syllabus and turns into a learning trajectory for a classroom community as well as its individual members (Keijzer & Terwel, 2001). Such a learning trajectory crucially depends on the resources available in the classroom, especially on the teacher being actively engaged in the entire teaching-learning process. In this light there is one obvious conclusion to be drawn: more attention needs to be paid to assisting the teacher in this complex process of guidance.

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Samenvatting

De leerling als ontwerper: Processen en effecten van een experimenteel programma voor het construeren van modellen in het reken-wiskundeonderwijs

Het thema van dit proefschrift is 'het ontwerpen van modellen in het reken-wiskundeonderwijs op de basisschool'. Het gezamenlijk construeren van modellen is bestudeerd door middel van een interventie waarin het effect van het 'in co-constructie zelf ontwerpen van modellen' vergeleken werd met directe instructie van modellen zoals ze meestal in het basisonderwijs toegepast worden: als kant en klare modellen die kinderen direct kunnen toepassen. In de literatuur zijn verschillende standpunten aan te treffen: sommige onderzoekers vinden dat men kinderen modellen moet aanreiken (Hattie, Biggs and Purdie, 1996; Perkins and Unger, 1999; Rosenshine, Meister & Chapman, 1996), terwijl andere onderzoekers juist vinden dat kinderen er meer van leren als ze zelf hun modellen leren ontwerpen (Davydov, 1988; diSessa, 1991; Meira, in press, en van Oers, in press).

De onderzoeksvraag was als volgt: wat zijn de effecten van een experimentele interventie voor het leren modelleren in contexten - waarbij leerlingen worden gezien als ontwerpers - op de leerprocessen en leerresultaten bij rekenen-wiskunde in het basisonderwijs, vergeleken met de leerresultaten van leerlingen in een controle interventie waarin leerlingen modellen kant en klaar aangereikt krijgen met de opdracht deze toe te passen in daartoe ontworpen situaties?

De hypothese was dat leerlingen van de ontwerpende, experimentele conditie beter in staat zouden zijn om complexe problemen in een transfer toets op te lossen. Dit zou komen door (1) de actieve(re) betrokkenheid van de leerlingen, en (2) de verbetering van het inzicht door het leren modellen te ontwerpen, een constructieve benadering van probleem oplossen. Daarentegen zouden de prestaties op curriculum gebonden opgaven niet anders hoeven zijn dan die van kinderen in de aanreikende, controle conditie, omdat deze opgaven in de lessen geoefend zijn en voor deze opgaven minder inzicht vereist is.

Deze onderzoeksvraag is bestudeerd in twee onderzoeken, een casestudie en een hoofdonderzoek, en de resultaten zijn vervolgens gerapporteerd in vier hoofdstukken. In twee hoofdstukken zijn de resultaten op een kwalitatieve wijze geanalyseerd, in twee andere hoofdstukken is een kwantitatieve benadering gekozen.

De casestudie vond plaats op één school, in twee groepen 7. Een van de groepen kreeg de aanreikende conditie toegewezen, de andere groep de ontwerpende. Van elke groep werd een gemiddelde leerling geselecteerd en van dichtbij gevolgd in hun leerprocessen. De resultaten van de case studie worden gepresenteerd in hoofdstuk 2. Video-opnames laten verschillen in aanpakken zien van de beide leerlingen. Over het algemeen blijkt de ontwerpende leerling beter met zelfontworpen modellen overweg te kunnen dan de leerling in de aanreikende conditie met de modellen die zij kreeg aangeboden.

Vervolgens is een grootschaliger onderzoek opgezet. Hierin participeerden 10 groepen, 10 leerkrachten en 238 kinderen uit groep 7. De helft van de groepen (117 leerlingen) werd aan de aanreikende conditie toegewezen, de overige kinderen deden mee aan de ontwerpende conditie (121 leerlingen). Alle leerlingen kregen gedurende drie weken één uur per dag rekenles over de onderwerpen procenten en grafieken. Open en complexe problemen in rijke contexten vormden de basis van de interventie. De kinderen in beide condities werkten aan dezelfde wiskundige inhoud, maar de manier waarop verschilde. In de aanreikende groep werkten de kinderen individueel met kant en klare modellen die door de leerkracht of het tekstboek werden aangeboden. In de ontwerpende groep daarentegen lag de nadruk op het in co-constructie zelf leren ontwerpen van modellen, waardoor een grotere diversiteit aan modellen naar voren kwam en besproken werd.

Het hoofdonderzoek wordt in drie hoofdstukken besproken. Hoofdstuk 3 bespreekt de resultaten op de posttest en hoofdstuk 4 de resultaten op de transfertest. In hoofdstuk 5 wordt dieper op de proceskant ingegaan. Op basis van de resultaten uit hoofdstuk 3 en 4 kunnen we concluderen dat kinderen in de ontwerpende groep significant hoger scoorden op zowel de posttest als de transfertest, dan kinderen uit de aanreikende groep. Leren hoe je zelf modellen moet ontwerpen om die later toe te kunnen passen in andere probleemsituaties en opgaven is dus een effectieve strategie.

In hoofdstuk 5 wordt in het bijzonder aandacht besteed aan de manier waarop het leren ontwerpen van modellen in zijn werk gaat, bijvoorbeeld welke processen zich voordoen en hoe interactieprocessen verlopen. Aan de hand van video-opnames zijn enkele kinderen gevolgd in hun ontwikkeling. Voor de analyse van het beeldmateriaal is een niveau indeling gebruikt die is geïnspireerd op Gravemeijers' niveau theorie. Deze theorie betreft het ontwikkelen van modellen van situatief naar algemeen, en die ontwikkeling kan worden doorlopen in vier niveaus. Voor dit onderzoek bleek vooral de overgang van het 'referential level' naar het 'general level' interessant. Gravemeijer spreekt hier ook wel van respectievelijk 'model-van' en 'model-voor'. Het oorspronkelijke, context specifieke model krijgt een meer formeel, abstract karakter en kan dan ook worden toegepast in diverse situaties. Het model wordt een 'entity of its own'. Dit nieuwe ontwikkelde model kan gebruikt worden als een 'tool' om op een formeler niveau wiskundig te redeneren. In onze videomaterialen vonden wij diverse voorbeelden van deze overgang van concreet naar abstract, wat zoveel wil zeggen als dat kinderen hun zelfbedachte modellen kunnen generaliseren en toepassen in nieuwe situaties.

De resultaten, zoals weergegeven in hoofdstuk 2 tot en met 5, laten duidelijk de positieve effecten van de experimentele ontwerpende conditie zien. De theoretische discussie en de resultaten van de beide onderzoeken indiceren dat het leren modelleren in co-constructie kan leiden tot een dieper inzicht in de betekenis en het gebruik van modellen in reken-wiskundeonderwijs.

Het verdient daarom aanbeveling om in het onderwijs expliciete aandacht te besteden aan het leren modelleren van situaties en die situaties op die manier te mathematiseren. Dat kan door leerlingen te stimuleren om hun eigen modellen te ontwikkelen en hen te confronteren met modellen van medeleerlingen en bestaande, formele modellen. Vanuit hun eigen actieve inbreng ontstaat op die manier een beter inzicht in de wijze waarop het formele model tot stand is gekomen, en ontwikkelen kinderen strategieën om in nieuwe wiskundige situaties zelf modellen te bedenken of zonodig aan te passen.

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Curriculum Vitae

Ivanka van Dijk werd op 31 juli 1974 geboren te Utrecht. Zij behaalde in 1992 het VWO-diploma aan het Niels Stensen College in Utrecht. Daarna begon zij aan haar studie orthopedagogiek aan de Universiteit van Amsterdam, met als afstudeerspecialisatie 'school- en leermoeilijkheden'. Deze studie werd in 1997 afgerond. Tijdens haar studie werkte zij, eerst als stagiaire en later als proefleider, mee aan het promotieonderzoek van Agnes Vosse naar effecten van tutorbegeleiding in reken-wiskundeonderwijs. Vervolgens was zij van 1998 tot 2002 als promovenda verbonden aan de afdeling Onderwijspedagogiek van de Vrije Universiteit te Amsterdam. Deze dissertatie is hiervan het eindproduct. Vanaf 2001 is zij parttime in dienst gekomen als schoolbegeleider bij de Onderwijsbegeleidingsgroep Kennemerland (OBGK) te Haarlem, waar zij thans nog werkzaam is. In deze functie houdt zij zich onder meer bezig met advisering en nascholing op het gebied van reken-wiskundeonderwijs, tutorbegeleiding, Daltononderwijs, hoogbegaafdheid en ICT.